

Practical Applications of Differentiation

Q1, (OCR 4752, Q10)

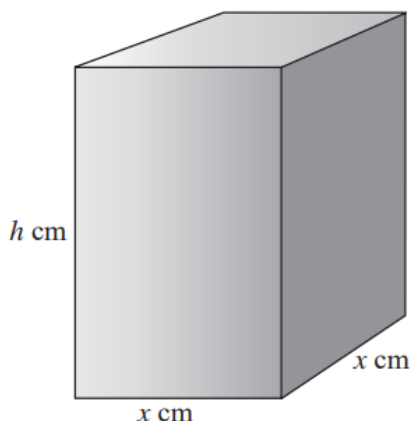


Fig. 10

Fig. 10 shows a solid cuboid with square base of side x cm and height h cm. Its volume is 120 cm^3 .

- (i) Find h in terms of x . Hence show that the surface area, $A \text{ cm}^2$, of the cuboid is given by $A = 2x^2 + \frac{480}{x}$. [3]

- (ii) Find $\frac{dA}{dx}$ and $\frac{d^2A}{dx^2}$. [4]

- (iii) Hence find the value of x which gives the minimum surface area. Find also the value of the surface area in this case. [5]

Q2, (Edexcel 6664, Jun 2015, Q9)

A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75 \pi \text{ cm}^3$.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

- (a) show that the cost of the polishing, £ C , is given by

$$C = 6\pi r^2 + \frac{300\pi}{r} \tag{4}$$

- (b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. [5]

- (c) Justify that the answer that you have obtained in part (b) is a minimum. [1]

Q3, (Edexcel 6664, Jan 2011, Q10)

The volume $V \text{ cm}^3$ of a box, of height $x \text{ cm}$, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$. (4)

(b) Hence find the maximum volume of the box. (4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. (2)

Q4, (Edexcel 6664, Jan 2012, Q8)

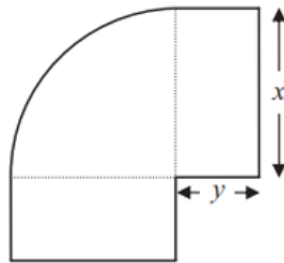


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \tag{3}$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \tag{3}$$

(c) Use calculus to find the minimum value of P . (5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre. (2)

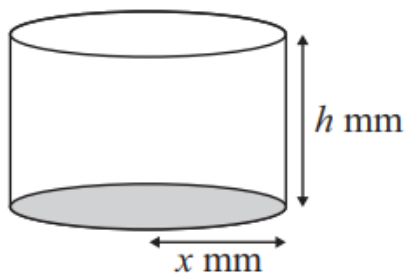


Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3 ,

(a) express h in terms of x , (1)

(b) show that the surface area, $A \text{ mm}^2$, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$ (3)

The manufacturer needs to minimise the surface area $A \text{ mm}^2$, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum. (5)

(d) Calculate the minimum value of A , giving your answer to the nearest integer. (2)

(e) Show that this value of A is a minimum. (2)

Q6, (OCR 4752, Jun 2011, Q11)

(i) The standard formulae for the volume V and total surface area A of a solid cylinder of radius r and height h are

$$V = \pi r^2 h \quad \text{and} \quad A = 2\pi r^2 + 2\pi r h.$$

Use these to show that, for a cylinder with $A = 200$,

$$V = 100r - \pi r^3. \quad [4]$$

(ii) Find $\frac{dV}{dr}$ and $\frac{d^2V}{dr^2}$. [3]

(iii) Use calculus to find the value of r that gives a maximum value for V and hence find this maximum value, giving your answers correct to 3 significant figures. [4]

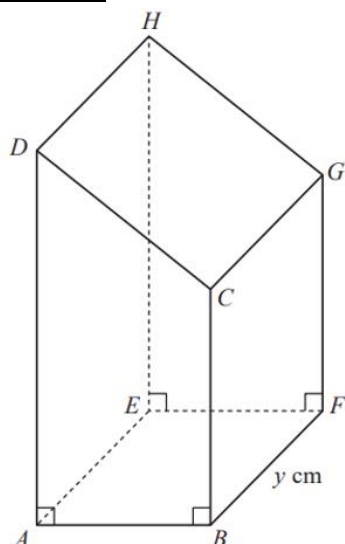


Figure 4

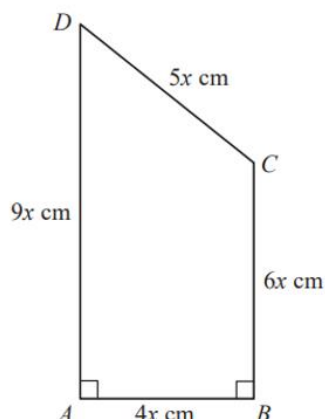


Figure 5

Figure 4 shows a closed letter box $ABFEHGCD$, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base $ABFE$ of the prism is a rectangle. The total surface area of the six faces of the prism is S cm².

The cross section $ABCD$ of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5. The angle $DAB = 90^\circ$ and the angle $ABC = 90^\circ$.

The volume of the letter box is 9600 cm³.

(a) Show that

$$y = \frac{320}{x^2} \tag{2}$$

(b) Hence show that the surface area of the letter box, S cm², is given by

$$S = 60x^2 + \frac{7680}{x} \tag{4}$$

(c) Use calculus to find the minimum value of S . (6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum. (2)