Polynomial Division and Factor Theorem Exam Questions MS (from OCR 4722)

Q1, (Jan 2006, Q8i)

(i)	$-2+k+1+6=0 \Rightarrow k=-5$	M1		For attempting f(–1)
		A1		For equating f(-1) to 0 and deducing the correct value of k **AG**
	OR	M1		Match coefficients and attempt k
		A1		Show k = -5
	OR	B2		Following division, state remainder is 0, hence $(x + 1)$ **is** a factor, hence k = -5
	EITHER: $(x+1)(2x^2-7x+6)$	B1		For correct leading term $2x^2$
		M1		For attempt at complete division by f(x) by $(x + 1)$ or equiv.
		A1		For completely correct quadratic factor
	$=(x+1)(x-2)(2x-3)$	A1		For all three factors correct
	OR: f(2) = 16 – 20 – 2 + 6 = 0	M1		For further relevant use of the factor theorem
	Hence $(x – 2)$ is a factor			
	Third factor is $(2x – 3)$	A1		For correct identification of factor $(x – 2)$
	Hence f(x) $=(x+1)(x-2)(2x-3)$	M1		For any method for the remaining factor
		A1	**6**	For all three factors correct

Q2, (Jan 2007, Q8)

(i)	-8 – 36 – 14 + 33 = -25	M1		Substitute x = -2, or attempt complete division by $(x + 2)$
		A1	**2**	Obtain –25, as final answer
(ii)	27 – 81 + 21 + 33 = 0 **A.G.**	B1	**1**	Confirm f(3) = 0, or equiv using division
(iii)	$x = 3$	B1		State x = 3 as a root at any point
	f(x) = $(x – 3)(x^2 – 6x – 11)$	M1		Attempt complete division by $(x – 3)$ or equiv
		A1		Obtain $x^2 – 6x + k$
		A1		Obtain completely correct quotient
	$x=\frac{6\pm\sqrt{36+44}}{2}$	M1		Attempt use of quadratic formula, or equiv, to find roots
	$=3\pm2\sqrt{5}$ or $3\pm\sqrt{20}$	A1	**6**	Obtain $3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$
			9	

Q3 (Jun 2007, Q9i,ii)

(i)	**(a)** f (-1) = – 1 + 6 – 1 – 4 = 0	B1	**1**	Confirm f(-1) = 0, through any method
	(b) x = -1	B1		State x = -1 at any point
	$f(x)=(x+1)(x^2+5x-4)$	M1		Attempt complete division by $(x + 1)$, or equiv
		A1		Obtain $x^2 + 5x + k$
		A1		Obtain completely correct quotient
	$x=\frac{-5\pm\sqrt{25+16}}{2}$	M1		Attempt use of quadratic formula, or equiv, find roots
	$x=\frac{1}{2}(-5\pm\sqrt{41})$	A1	**6**	Obtain $\frac{1}{2}(-5\pm\sqrt{41})$

Q4 (Jun 2008, Q4)

(i) $f(3) = 27a - 36 - 21a + 12 = 0$
$6a = 24$
$a = 4$

M1* Attempt f(3)
M1d* Equate attempt at f(3) to 0 and attempt to solve
A1 Obtain $a = 4$

OR

M1* Attempt complete division / matching coeffs
M1d* Equate remainder to 0
A1 Obtain $a = 4$
3

(ii) $f(-2) = -32 - 16 + 56 + 12$
$= 20$

M1 Attempt f(-2)
A1√ Obtain 20 (or $6a - 4$, following their a)
2

Q5 (Jun 2009, Q7)

(i) $f(-2) = -16 + 36 - 22 - 8$ $= -10$	M1 A1	**2**	Attempt f(-2), or equiv Obtain -10
(ii) $f(½) = ¼ + 2¼ + 5½ - 8 = 0$ AG	M1 A1	**2**	Attempt f(½) (no other method allowed) Confirm f(½) = 0, extra line of working required
(iii) $f(x) = (2x - 1)(x^2 + 5x + 8)$	M1 A1 A1	**3**	Attempt complete division by $(2x - 1)$ or $(x - ½)$ or equiv Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$ State $(2x - 1)(x^2 + 5x + 8)$ or $(x - ½)(2x^2 + 10x + 16)$
(iv) $f(x)$ has one real root ($x = ½$) because $b^2 - 4ac = 25 - 32 = -7$ hence quadratic has no real roots as $-7 < 0$,	B1√ B1√	**2**	State 1 root, following their quotient, ignore reason Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at (-2.15, -9.9)

9

Q6 (Jan 2012, Q5)

(i)	$f(3) = 54 + 27 - 51 + 6$ $= 36$	**M1** **A1** **[2]**	Attempt f(3) Obtain 36

(ii)		$f(x) = (x-2)(2x^2+7x-3)$	**B1**	State or imply that $(x-2)$ is a factor
			M1	Attempt full division, or equiv, by $(x \pm 2)$
			A1	Obtain $2x^2$ and at least one other correct term
			A1	Obtain $(x-2)(2x^2+7x-3)$
			[4]	
(iii)		$b^2 - 4ac = 73$ > 0 hence 3 roots	**M1**	Attempt explicit numerical calculation to find number of roots of quadratic
			A1ft	State 3 roots (√ their quotient) Condone no explicit check for repeated roots
			[2]	

Q7 (Jun 2013, Q9)

(i)		$f(2) = 32 - 14 - 3 = 15$	M1	Attempt f(2) or equiv
			A1	Obtain 15
			[2]	
(ii)		$f(-\frac{1}{2}) = -\frac{1}{2} + \frac{7}{2} - 3 = 0$ **AG**	B1	Confirm $f(-\frac{1}{2}) = 0$, with at least one line of working
		$f(x) = (2x + 1)(2x^2 - x - 3)$	M1	Attempt complete division by $(2x + 1)$, or another correct factor
		$= (2x + 1)(2x - 3)(x + 1)$	A1	Obtain $2x^2$ and one other correct term
			A1	Obtain fully correct quotient of $2x^2 - x - 3$
			M1	Attempt to factorise their quadratic quotient from division attempt by correct factor
			A1	Obtain $(2x + 1)(2x - 3)(x + 1)$

(iii)		$2\cos\theta + 1 = 0$ $\quad \cos\theta + 1 = 0$ $2\cos\theta - 3 = 0$		M1*	Identify relationship between factors of f($\cos\theta$) and factors of f(x)
		$\cos\theta = {}^{-1}/_{2}$ $\quad \cos\theta = -1$ $\cos\theta = {}^{3}/_{2}$		M1d*	Attempt to solve $\cos\theta = k$ at least once
		$\theta =$ 120, 240 $\quad$ 180		A1	Obtain at least 2 correct angles
				A1	Obtain all 3 correct angles

Q8 (Jun 2014, Q7)

(i)		$f(-2) = 12 - 22(-2) + 9(-2)^2 - (-2)^3$ $= 12 + 44 + 36 + 8$	M1	Attempt f(–2) or equiv
		$= 100$	A1 **[2]**	Obtain 100
(ii)		$f(3) = 12 - 66 + 81 - 27 = 0$	B1	Attempt f(3), and show = 0

(iii)		$f(x) = (3 - x)(x^2 - 6x + 4)$	M1	Attempt complete division by $(3 - x)$ or $(x - 3)$, or equiv
			A1	Obtain $x^2 - 6x + 4$ or $-x^2 + 6x - 4$
			A1	Obtain $(3 - x)(x^2 - 6x + 4)$ or $(x - 3)(-x^2 + 6x - 4)$
			[3]	
(iv)		$x = 3$	B1	State $x = 3$
		$x = 3 \pm \sqrt{5}$	M1	Attempt to find roots of quadratic quotient
			A1	Obtain $x = 3 \pm \sqrt{5}$
			[3]	