Integration Exam Questions MS (From OCR 4722 unless otherwise stated)

Q1, (Jun 2007, Q6)

(a) **(i)** $\int x^3 - 4x = \frac{1}{4}x^4 - 2x^2 + c$	M1		Expand and attempt integration
	A1		Obtain $\frac{1}{4}x^4 - 2x^2$ (A0 if $\int$ or dx still present)
	B1	**3**	$+ c$ (mark can be given in (b) if not gained here)
(ii) $\left[\frac{1}{4}x^4 - 2x^2\right]_1^6$	M1		Use limits correctly in integration attempt (ie F(6) – F(1))
$=(324 - 72) - (\frac{1}{4} - 2)$ $= 253\frac{3}{4}$	A1	**2**	Obtain $253\frac{3}{4}$ (answer only is M0A0)
(b) $\int 6x^{-3}\mathrm{d}x = -3x^{-2} + c$	B1		Use of $\frac{1}{x^3} = x^{-3}$
	M1		Obtain integral of the form kx^{-2}
	A1	**3**	Obtain correct $-3x^{-2}$ $(+ c)$ (A0 if $\int$ or dx still present, but only penalise once in question)
		8	

Q2, (Jan 2008, Q7)

(i) Some of the area is below the x-axis	B1	**1**	Refer to area / curve below x-axis or 'negative area'…
(ii)	M1		Attempt integration with any one term correct
	A1		Obtain $\frac{1}{3}x^3 - \frac{3}{2}x^2$
$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - (0 - 0)$	M1		Use limits 3 (and 0) – correct order / subtraction
$= -4\frac{1}{2}$	A1		Obtain (-)$4\frac{1}{2}$
$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
$= 8\frac{2}{3}$	A1		Obtain $8\frac{2}{3}$ (allow 8.7 or better)
Hence total area is $13\frac{1}{6}$	A1	**7**	Obtain total area as $13\frac{1}{6}$, or exact equiv
			SR: if no longer $\int f(x)\mathrm{d}x$, then B1 for using [0, 3] and [3, 5]
		8	

Q3, (Jan 2009, Q1)

(i) $\int (x^3 + 8x - 5)\mathrm{d}x = \frac{1}{4}x^4 + 4x^2 - 5x + c$	M1		Attempt integration – increase in power for at least 2 terms
	A1		Obtain at least 2 correct terms
	A1	**3**	Obtain $\frac{1}{4}x^4 + 4x^2 - 5x + c$ (and no integral sign or dx)
(ii) $\int 12x^{\frac{1}{2}}\mathrm{d}x = 8x^{\frac{3}{2}} + c$	B1		State or imply $\sqrt{x} = x^{\frac{1}{2}}$
	M1		Obtain $kx^{\frac{3}{2}}$
	A1	**3**	Obtain $8x^{\frac{3}{2}} + c$ (and no integral sign or dx) (only penalise lack of + c, or integral sign or dx once)
		6	

Q4, (Jun 2011, Q2)

(i)	$\int\left(6x^{\frac{1}{2}}-1\right)dx = 4x^{\frac{3}{2}} - x + c$	**M1**		Obtain $kx^{\frac{3}{2}}$
		A1		Obtain $4x^{\frac{3}{2}}$
		B1	**3**	Obtain $-x$ (don't penalise lack of $+c$)
(ii)	$y = 4x^{\frac{3}{2}} - x + c$ $17 = 32 - 4 + c \Rightarrow c = -11$ hence $y = 4x^{\frac{3}{2}} - x - 11$	**M1***		State or imply $y =$ their integral from (i)
		M1d*		Attempt to find c using (4, 17)
		A1	**3**	Obtain $y = 4x^{\frac{3}{2}} - x - 11$

Q5, (Jun 2015, Q5)

$^{dy}/_{dx} = 6x^{0.5} + c$	M1*	Attempt integration
	A1	Obtain $6x^{0.5}$ (allow no + c)
$5 = 12 + c$	M1d*	Attempt to use $x = 4$, gradient = 5
$c = -7$	A1	Rearrange to obtain $c = -7$
$y = 4x^{1.5} - 7x + k$	M1 dd*	Attempt second integration
$1 = 32 - 28 + k$, hence $k = -3$	M1 ddd*	Attempt to find k using (4, 1)
$y = 4x^{1.5} - 7x - 3$	A1	Obtain $y = 4x^{1.5} - 7x - 3$
	[7]	

Q6, (Jan 2009, Q4)

4 $\int_{-2}^{2}(x^4+3)\,dx = \left[\frac{1}{5}x^5+3x\right]_{-2}^{2}$	M1		Attempt integration – increase of power for at least 1 term
	A1		Obtain correct $\frac{1}{5}x^5+3x$
$= (\frac{32}{5}+6)-(\frac{-32}{5}-6)$	M1		Use limits (any two of –2, 0, 2), correct order/subtraction
$= 24\frac{4}{5}$	A1		Obtain $24\frac{4}{5}$
area of rectangle = 19 x 4	B1		State or imply correct area of rectangle
hence shaded area = 76 – $24\frac{4}{5}$	M1		Attempt correct method for shaded area
$= 51\frac{1}{5}$	A1	**7**	Obtain $51\frac{1}{5}$ aef such as 51.2, $\frac{256}{5}$
OR			
Area = 19 – (x^4+3)	M1		Attempt subtraction, either order
$= 16-x^4$	A1		Obtain $16-x^4$ (not from $x^4+3=19$)
$\int_{-2}^{2}(16-x^4)\,dx = \left[16x-\frac{1}{5}x^5\right]_{-2}^{2}$	M1		Attempt integration
	A1		Obtain $\pm\left(16x-\frac{1}{5}x^5\right)$
$= (32-\frac{32}{5})-(-32-\frac{-32}{5})$	M1		Use limits – correct order / subtraction
$= 51\frac{1}{5}$	A1		Obtain $\pm 51\frac{1}{5}$
	A1		Obtain $51\frac{1}{5}$ only, no wrong working
	7		

Q7, (Jan 2011, Q9)

(i)	$f(3) = -108 + 81 + 30 - 3 = 0$ hence $(x - 3)$ is a factor	**B1**		Show that $f(3) = 0$, detail required
		B1	**2**	State $(x - 3)$ as factor (allow $(3 - x)$ as the factor)
(ii)	$f(x) = (x - 3)(-4x^2 - 3x + 1)$ or $f(x) = (3 - x)(4x^2 + 3x - 1)$ or $f(x) = (x + 1)(-4x^2 + 13x - 3)$ or $f(x) = (-x - 1)(4x^2 - 13x + 3)$ or $f(x) = (1 - 4x)(x^2 - 2x - 3)$ or $f(x) = (4x - 1)(-x^2 + 2x + 3)$	**M1**		Attempt complete division by $(x - 3)$, or equiv (allow division by $(3 - x)$)
		A1		Obtain $-4x^2 - 3x + c$ or $-4x^2 + bx + 1$ (or the negative of these if dividing by $(3 - x)$)
		A1	**3**	Obtain $(x - 3)(-4x^2 - 3x + 1)$ (or $(3 - x)(4x^2 + 3x - 1)$)
(iii)	$-4x^2 - 3x + 1 = 0$ $(1 - 4x)(x + 1) = 0$ $x = ¼ , x = -1$	**M1**		Attempt to solve quadratic
		A1	**2**	Obtain $(¼ , 0)$, $(-1, 0)$

(iv)	$\int f(x)dx = -x^4 + 3x^3 + 5x^2 - 3x$	B1		Obtain $-x^4 + 3x^3 + 5x^2 - 3x$
	$F(3) - F(\frac{1}{4}) = (36) - (^{-101}/_{256}) = 36\,^{101}/_{256}$ $F(\frac{1}{4}) - F(-1) = (^{-101}/_{256}) - (4) = -4\,^{101}/_{256}$	M1*		Attempt F(3) – F(¼) or F(¼) – F(-1)
		A1		Obtain at least one correct area, including decimal equivs
		M1d*		Attempt full method to find total area including dealing correctly with negative area
	Hence area = $36\,^{101}/_{256} + 4\,^{101}/_{256} = 40\,^{101}/_{128}$	A1	5	Obtain $40\,^{101}/_{128}$ or $^{5221}/_{128}$ or 40.8
			12	

Q8, (Jun 2013, Q7)

(i)		$\int_1^4 \left(x^{\frac{3}{2}} - 1\right)dx = \left[\frac{2}{5}x^{\frac{5}{2}} - x\right]_1^4$	M1	Attempt integration
			A1	Obtain fully correct integral
		$= (12.8 - 4) - (0.4 - 1)$	M1	Attempt correct use of limits
		$= 9^2/_5$ **AG**	A1	Obtain $9^2/_5$
			[4]	
(ii)		$m = {}^3/_2 \times \sqrt{4} = 3$	M1*	Attempt to find gradient at (4, 7) using differentiation
		$y = 3x - 5$	M1d*	Attempt to find point of intersection of tangent with x-axis or attempt to find base of triangle
		tangent crosses x-axis at $({}^5/_3, 0)$	A1	Obtain $x = {}^5/_3$ as pt of intersection or obtain ${}^7/_3$ as base of triangle
		area of triangle $= {}^1/_2 \times (4 - {}^5/_3) \times 7$ $= 8^1/_6$	M1d**	Attempt complete method to find shaded area
		shaded area $= 9^2/_5 - 8^1/_6 = 1^7/_{30}$	A1	Obtain $1^7/_{30}$, or exact equiv
			[5]	

Q9, (OCR H230/01, Practice Papers Set 1, Q6)

(i)		$\frac{x^4}{4} - \frac{x^3}{3} - x^2$	**M1**	**1.1**	Increase at least two indices by 1
			A1	**1.1**	At least 2 terms correct
		$\frac{x^4}{4} - \frac{x^3}{3} - x^2 + c$	**A1**	**1.2**	All correct with $+ c$
			[3]		
(ii)		**DR** $x^3 - x^2 - 2x = 0$	**M1**	**1.1a**	
		$x = 0$ or 2 (or -1)	**A1**	**1.1**	Allow just $x = 0$ or 2
		$\int_0^2 (x^3 - x^2 - 2x)dx$	**M1**	**1.1**	
		$(= -\frac{8}{3})$ Hence area $= \frac{8}{3}$	**A1**	**2.2a**	
			[4]		