

Differentiation From First Principles Exam Questions MS (From OCR MEI 4752 unless otherwise stated)

Q1, (Jun 2009, Q12)

| | | | | |
|------------|--|----------------|---|---|
| i | 6.1 | 2 | M1 for $\frac{(3.1^2 - 7) - (3^2 - 7)}{3.1 - 3}$ o.e. | 2 |
| ii | $\frac{((3+h)^2 - 7) - (3^2 - 7)}{h}$ numerator = $6h + h^2$ $6 + h$ | M1 M1 A1 | s.o.i. | 3 |
| iii | as h tends to 0, grad. tends to 6 o.e. f.t.from " $6+h$ " | M1 A1 | | 2 |
| iv | $y - 2 = "6" (x - 3)$ o.e. $y = 6x - 16$ | M1 A1 | 6 may be obtained from $\frac{dy}{dx}$ | 2 |
| v | At P, $x = 16/6$ o.e. or ft At Q, $x = \sqrt{7}$ 0.021 cao | M1 M1 A1 | | 3 |

Q2, (Jan 2007, Q5)

| | | | |
|--|------|--|---|
| (i) -0.93, -0.930, -0.9297... | 2 | M1 for grad = $(1 - \text{their } y_B)/(2 - 2.1)$ if M0, SC1 for 0.93 | |
| (ii) answer strictly between 1.91 and 2 or 2 and 2.1 | B1 | don't allow 1.9 recurring | |
| (iii) $y' = -8/x^3$, gradient = -1 | M1A1 | | 5 |

Q3, (Jun 2010, Q10)

| | | | |
|----------------------------|---|-----------|--|
| (i) | $\frac{dy}{dx} = 4x^3$ | M1 | i.s.w. |
| | when $x = 2$, $\frac{dy}{dx} = 32$ s.o.i. | A1 | |
| | when $x = 2$, $y = 16$ s.o.i. | B1 | |
| | $y = 32x - 48$ c.a.o. | A1 | |
| (ii) | 34.481 | 2 | M1 for $\frac{2.1^4 - 2^4}{0.1}$ |
| (iii) (A) | $16 + 32h + 24h^2 + 8h^3 + h^4$ c.a.o. | 3 | B2 for 4 terms correct B1 for 3 terms correct |
| (iii) (B) | $32 + 24h + 8h^2 + h^3$ or ft | 2 | B1 if one error |
| (iii) (C) | as $h \rightarrow 0$, result \rightarrow their 32 from (iii) (B) | 1 | |
| | gradient of tangent is limit of gradient of chord | 1 | |

Q4, (OCR H230/02, Sample Question Paper, Q7)

| | | | |
|---|------------|------------|---|
| $f(x+h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$ | M1 | 1.1 | Attempt at expansion with product of powers of x and h summing to 4 and some attempt at coefficients, not necessarily correct |
| $\frac{f(x+h) - f(x)}{h} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$ | M1 | 1.1 | Attempt $\frac{f(x+h) - f(x)}{h}$ |
| $= 4x^3 + 6x^2h + 4xh^2 + h^3$ | A1 | 1.1 | Allow at most two errors |
| As $h \rightarrow 0$ all the terms in h tend to zero. | A1 | 2.4 | Accept some indication that as h tends to 0, the terms involving h vanish and leave $4x^3$ |
| Therefore $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 4x^3$ | E1 | 2.1 | Award for good use of language, and of limit and function notation |
| | [5] | | |

Q5, (Jun 2016, Q10)

| | | | |
|------------|--|------------|------------------------------|
| (i) | $\frac{(5.1^2 - 10.2) - (5^2 - 10)}{5.1 - 5}$ oe | M1 | condone omission of brackets |
| | 8.1 | A1 | |
| | | [2] | |

| | | | |
|--------------|---|--|---|
| (ii) | $\frac{(5+h)^2 - 2(5+h) - \text{their } 15}{h} \text{ oe}$ <p>25 + 10h + h² - 10 - 2h oe seen</p> <p>numerator is 8h + h²</p> <p>8 + h isw</p> | <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p> | <p>condone omission of brackets</p> <p>allow one sign error</p> |
| (iii) | <p>$h \rightarrow 0$</p> <p>their 8</p> | <p>M1</p> <p>A1</p> <p>[2]</p> | <p>may be embedded; allow eg “tends to 0”</p> <p>FT their $k + h$ from part (ii)</p> |
| (iv) | <p>$y = 8x - 25$ isw</p> <p>non-zero numerical value for x-intercept on their straight line found</p> <p>[x =] 3.125 oe</p> <p>$\frac{1}{2} \times$ their non-zero y-intercept \times their $\frac{25}{8}$</p> <p>$\frac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw</p> | <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p> | <p>or $y - 15 = 8(x - 5)$ isw or $y = 8x + c$ and $c = -25$ stated isw</p> <p>may be embedded in calculation for area</p> <p>condone arithmetic slips in finding values of intercepts</p> <p>accept rounded to 1 dp or better for A1; but A0 if final answer negative</p> |

Q6, (OCR 4721, Jun 2016, Q8)

| | | | |
|--------------|--|--|---|
| (i) | $y_1 = 50, y_2 = 2(5+h)^2$ $\frac{(50+20h+2h^2) - 50}{(5+h) - 5}$ $20 + 2h$ | <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> | <p>Finds y coordinates at 5 and 5 + h</p> <p>Correct method to find gradient of a line segment; at least 3/4 values correct</p> <p>Fully correct working to give answer AG</p> |
| (ii) | <p>e.g. “As h tends to zero, the gradient will be 20”</p> | <p>B1</p> <p>[1]</p> | <p>Indicates understanding of limit See Appendix 2 for examples</p> |
| (iii) | <p>Gradient of normal = $-\frac{1}{20}$</p> <p>$y - 50 = -\frac{1}{20}(x - 5), x = 0$</p> <p>50¼</p> | <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> | <p>Gradient of line must be numerical negative reciprocal of their gradient at A through their A</p> <p>Correct coordinate in any form e.g. $\frac{201}{4}, \frac{1005}{20}$</p> |

Q7, (Edexcel 8MA0/01, Sample Assessment 1, Q6)

| | | |
|--|-----|------|
| Considers $\frac{3(x+h)^2 - 3x^2}{h}$ | B1 | 2.1 |
| Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$ | M1 | 1.1b |
| So gradient = $\frac{6xh + 3h^2}{h} = 6x + 3h$ or $\frac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$ | A1 | 1.1b |
| States as $h \rightarrow 0$, gradient $\rightarrow 6x$ so in the limit derivative = $6x^*$ | A1* | 2.5 |

(4 marks)