Differentiation Exam Questions (From OCR MEI 4752 unless otherwise stated)

Q1, (Jan 2006, Q6)

A curve has gradient given by $\frac{dy}{dx} = x^2 - 6x + 9$. Find $\frac{d^2y}{dx^2}$.

Show that the curve has a stationary point of inflection when $x = 3$. [4]

Q2, (Jan 2007, Q1)

Differentiate $6x^{\frac{5}{2}} + 4$. [2]

Q3, (Jan 2009, Q7)

Differentiate $4x^2 + \frac{1}{x}$ and hence find the x-coordinate of the stationary point of the curve $y = 4x^2 + \frac{1}{x}$. **[5]**

Q4 (Jun 2007, Q9)

The equation of a cubic curve is $y = 2x^3 - 9x^2 + 12x - 2$.

(i) Find $\frac{dy}{dx}$ and show that the tangent to the curve when $x = 3$ passes through the point $(-1, -41)$. [5]

(ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]

(iii) Sketch the curve, given that the only real root of $2x^3 - 9x^2 + 12x - 2 = 0$ is $x = 0.2$ correct to 1 decimal place. [3]

Q5, (Jun 2009, Q6)

Use calculus to find the x-coordinates of the turning points of the curve $y = x^3 - 6x^2 - 15x$.

Hence find the set of values of x for which $x^3 - 6x^2 - 15x$ is an increasing function. **[5]**

Q6, (Jan 2013, Q6)

Differentiate $2x^3 + 9x^2 - 24x$. Hence find the set of values of x for which the function $f(x) = 2x^3 + 9x^2 - 24x$ is increasing. **[4]**

Q7, (Jun 2009, Q12)

(i) Calculate the gradient of the chord joining the points on the curve $y = x^2 - 7$ for which $x = 3$ and $x = 3.1$. **[2]**

(ii) Given that $f(x) = x^2 - 7$, find and simplify $\dfrac{f(3+h) - f(3)}{h}$. **[3]**

(iii) Use your result in part **(ii)** to find the gradient of $y = x^2 - 7$ at the point where $x = 3$, showing your reasoning. **[2]**

(iv) Find the equation of the tangent to the curve $y = x^2 - 7$ at the point where $x = 3$. **[2]**

(v) This tangent crosses the x-axis at the point P. The curve crosses the positive x-axis at the point Q. Find the distance PQ, giving your answer correct to 3 decimal places. **[3]**

Q8, (Jan 2006, Q11)

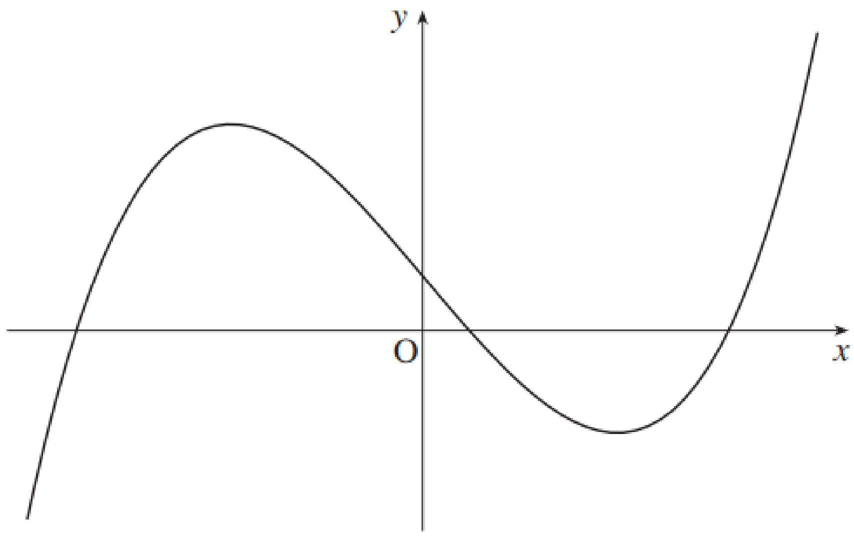

Fig. 11

The equation of the curve shown in Fig. 11 is $y = x^3 - 6x + 2$.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Find, in exact form, the range of values of x for which $x^3 - 6x + 2$ is a decreasing function. [3]

(iii) Find the equation of the tangent to the curve at the point $(-1, 7)$.

Find also the coordinates of the point where this tangent crosses the curve again. [6]

Q9, (Jan 2007, Q5)

A is the point $(2, 1)$ on the curve $y = \frac{4}{x^2}$.

B is the point on the same curve with x-coordinate 2.1.

(i) Calculate the gradient of the chord AB of the curve. Give your answer correct to 2 decimal places. [2]

(ii) Give the x-coordinate of a point C on the curve for which the gradient of chord AC is a better approximation to the gradient of the curve at A. [1]

(iii) Use calculus to find the gradient of the curve at A. [2]

Q10, (Jun 2014, Q11)

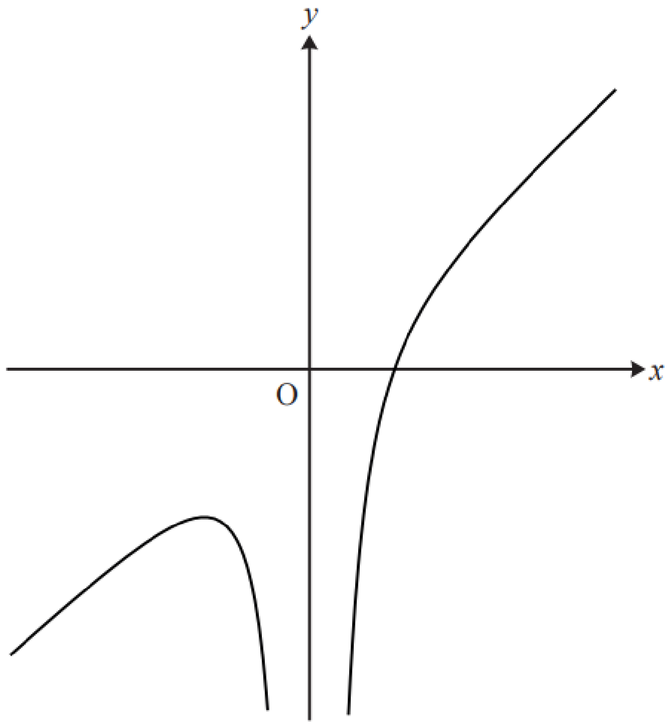

Fig. 11

Fig. 11 shows a sketch of the curve with equation $y = x - \frac{4}{x^2}$.

(i) Find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} = -\frac{24}{x^4}$. **[3]**

(ii) Hence find the coordinates of the stationary point on the curve. Verify that the stationary point is a maximum. **[5]**

(iii) Find the equation of the normal to the curve when $x = -1$. Give your answer in the form $ax + by + c = 0$. **[5]**

Q11, (Jun 2016, Q10)

(i) Calculate the gradient of the chord of the curve $y = x^2 - 2x$ joining the points at which the values of x are 5 and 5.1. **[2]**

(ii) Given that $f(x) = x^2 - 2x$, find and simplify $\frac{f(5+h) - f(5)}{h}$. **[4]**

(iii) Use your result in part (ii) to find the gradient of the curve $y = x^2 - 2x$ at the point where $x = 5$, showing your reasoning. **[2]**

(iv) Find the equation of the tangent to the curve $y = x^2 - 2x$ at the point where $x = 5$.

Find the area of the triangle formed by this tangent and the coordinate axes. **[5]**

Q12, (OCR 4721, Jun 2015, Q9)

The curve $y = 2x^3 - ax^2 + 8x + 2$ passes through the point B where $x = 4$.

(i) Given that B is a stationary point of the curve, find the value of the constant a. **[5]**

(ii) Determine whether the stationary point B is a maximum point or a minimum point. **[2]**

(iii) Find the x-coordinate of the other stationary point of the curve. **[3]**

Q13, (OCR 4721, Jun 2016, Q8)

A curve has equation $y = 2x^2$. The points A and B lie on the curve and have x-coordinates 5 and $5+h$ respectively, where $h > 0$.

(i) Show that the gradient of the line AB is $20 + 2h$. **[3]**

(ii) Explain how the answer to part **(i)** relates to the gradient of the curve at A. **[1]**

(iii) The normal to the curve at A meets the y-axis at the point C. Find the y-coordinate of C. **[3]**

Q14, (OCR 4721, Jun 2016, Q11)

The curve $y = 4x^2 + \frac{a}{x} + 5$ has a stationary point. Find the value of the positive constant a given that the y-coordinate of the stationary point is 32. **[8]**

Q15, (OCR 4721, Jun 2017, Q11)

The normal to the curve $y = \frac{k}{x^2}$ at the point where $x = -3$ is parallel to the line $\frac{1}{2}y = 2 + 3x$.

(i) Determine the value of the constant k. **[6]**

(ii) Find the equation of the normal where $x = -3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

Q16, (Jun 2010, Q10)

(i) Find the equation of the tangent to the curve $y = x^4$ at the point where $x = 2$. Give your answer in the form $y = mx + c$. **[4]**

(ii) Calculate the gradient of the chord joining the points on the curve $y = x^4$ where $x = 2$ and $x = 2.1$. **[2]**

(iii) (A) Expand $(2 + h)^4$. **[3]**

(B) Simplify $\dfrac{(2 + h)^4 - 2^4}{h}$. **[2]**

(C) Show how your result in part **(iii)** (B) can be used to find the gradient of $y = x^4$ at the point where $x = 2$. **[2]**

Q17, (OCR H230/02, Sample Question Paper, Q7)

Differentiate $f(x) = x^4$ from first principles. **[5]**
