Differentiation Exam Questions MS (From OCR MEI 4752 unless otherwise stated)

Q1, (Jan 2006, Q6)

$y'' = 2x - 6$	B1		
$y'' = 0$ at $x = 3$	B1		
$y' = 0$ at $x = 3$	B1		
showing y' does not change sign	E1	or that y'' changes sign	4

Q2, (Jan 2007, Q1)

$\frac{5}{2} \times 6x^{\frac{3}{2}}$	1+1	- 1 if extra term	2

Q3, (Jan 2009, Q7)

$8x - x^{-2}$ o.e.	2	B1 each term	
their $\frac{dy}{dx} = 0$	M1	s.o.i.	5
correct step	DM1	s.o.i.	
$x = ½$ c.a.o.	A1		

Q4 (Jun 2007, Q9)

i	$y' = 6x^2 - 18x + 12$	M1	condone one error	
	$= 12$	M1	subst of $x = 3$ in their y'	
	$y = 7$ when $x = 3$	B1		
	tgt is $y - 7 = 12(x - 3)$	M1	f.t. their y and y'	
	verifying $(-1, -41)$ on tgt	A1	or B2 for showing line joining (3, 7) and $(-1, -41)$ has gradient 12	5
ii	$y' = 0$ soi	M1	Their y'	
	quadratic with 3 terms	M1	Any valid attempt at solution	
	$x = 1$ or 2	A1	or A1 for (1, 3) and A1 for (2,2) marking	
	$y = 3$ or 2	A1	to benefit of candidate	4
iii	cubic curve correct orientation touching x- axis only at (0.2,0)	G1		
	max and min correct	G1	f.t.	
	curve crossing y axis only at -2	G1		3

$y' = 3x^2 - 12x - 15$	M1	for two terms correct	
use of $y' = 0$, s.o.i. ft	M1		
$x = 5, -1$ c.a.o.	A1		
$x < -1$ or $x > 5$ f.t.	A1		
	A1		5

Q5, (Jun 2009, Q6)

$y' = 3x^2 - 12x - 15$	M1	for two terms correct	
use of $y' = 0$, s.o.i. ft	M1		
$x = 5, -1$ c.a.o.	A1		
$x < -1$ or $x > 5$ f.t.	A1		
	A1		5

Q6, (Jan 2013, Q6)

$6x^2 + 18x - 24$	B1	
their $6x^2 + 18x - 24 = 0$ or > 0 or ≥ 0	M1	
-4 and $+1$ identified oe	A1	
$x < -4$ and $x > 1$ cao	A1	or $x \leq -4$ and $x \geq 1$
	[4]	

Q7, (Jun 2009, Q12)

i	6.1	2	M1 for $\frac{(3.1^2-7)-(3^2-7)}{3.1-3}$ o.e.	2
ii	$\frac{((3+h)^2-7)-(3^2-7)}{h}$	M1	s.o.i.	
	numerator $= 6h + h^2$	M1		
	$6 + h$	A1		3
iii	as h tends to 0,	M1		
	grad. tends to 6 o.e. f.t.from "6"+h	A1		2
iv	$y - 2 =$ "6" $(x - 3)$ o.e.	M1		
	$y = 6x - 16$	A1	6 may be obtained from $\frac{dy}{dx}$	2
v	At P, $x = 16/6$ o.e. or ft	M1		
	At Q, $x = \sqrt{7}$	M1		
	0.021 cao	A1		3

Q8, (Jan 2006, Q11)

i	$3x^2 - 6$	2	1 if one error	2
ii	$-\sqrt{2} < x < \sqrt{2}$	3	M1 forusing their $y' = 0$ B1 f.t. forboth roots found	3
iii	subst $x = -1$ in their y' [$=-3$] $y = 7$ when $x = -1$ $y + 3x = 4$ $x^3 - 6x + 2 = -3x + 4$ $(2, -2)$ c.a.o.	B1 M1 A1 M1 A1,A1	f.t. f.t. 3 terms f.t.	6

Q9, (Jan 2007, Q5)

(i) −0.93, -0.930, -0.9297… (ii) answer strictly between 1.91 and 2 or 2 and 2.1 (iii) $y' = -8/x^3$, gradient = −1	2 B1 M1A1	M1 for grad = (1 – their y_B)/(2 − 2.1) if M0, SC1 for 0.93 don't allow 1.9 recurring	5

Q10, (Jun 2014, Q11)

(i)		$y' = 1 + 8x^{-3}$	M2	M1 for just $8x^{-3}$ or $1 - 8x^{-3}$
		$y'' = -24x^{-4}$ oe	A1	
			[3]	
(ii)		their $y' = 0$ soi	M1	
		$x = -2$	A1	A0 if more than one x-value
		$y = -3$	A1	A0 if more than one y-value
		substitution of $x = -2$: $\dfrac{-24}{(-2)^4}$	M1	or considering signs of gradient either side of -2 with negative x-values
		< 0 or $= -1.5$ oe correctly obtained isw	A1	signs for gradients identified to verify maximum
			[5]	
(iii)		$y = -5$ soi	B1	
		substitution of $x = -1$ in their y'	M1	may be implied by -7
		grad normal $= {}^{-1}/_{\text{their } -7}$	M1*	may be implied by eg ${}^{1}/_{7}$
		$y -$ their$(-5) =$ their ${}^{1}/_{7}(x - -1)$	M1dep*	or their $(-5) =$ their ${}^{1}/_{7} \times (-1) + c$
		$-x + 7y + 34 = 0$ oe	A1	allow eg $y - \frac{1}{7}x + \frac{34}{7} = 0$
			[5]	

Q11, (Jun 2016, Q10)

(i)		$\dfrac{(5.1^2 - 10.2) - (5^2 - 10)}{5.1 - 5}$ oe	**M1**	condone omission of brackets
		8.1	**A1**	
			[2]	

(ii)		$\dfrac{(5+h)^2-2(5+h)-\text{ their } 15}{h}$ oe	**M1**	condone omission of brackets
		$25 + 10h + h^2 - 10 - 2h$ oe seen	**M1**	allow one sign error
		numerator is $8h + h^2$	**A1**	
		$8 + h$ isw	**A1** **[4]**	
(iii)		$h \to 0$	**M1**	may be embedded; allow eg "tends to 0"
		their 8	**A1** **[2]**	**FT** their $k + h$ from part (ii)
(iv)		$y = 8x - 25$ isw	**B1**	or $y - 15 = 8(x - 5)$ isw or $y = 8x + c$ and $c = -25$ stated isw
		non-zero numerical value for x-intercept on their straight line found	**M1**	
		[$x =$] 3.125 oe	**A1**	may be embedded in calculation for area
		$\frac{1}{2} \times$ their non-zero y-intercept $\times$ their $\frac{25}{8}$	**M1**	condone arithmetic slips in finding values of intercepts
		$\frac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw	**A1** **[5]**	accept rounded to 1 dp or better for **A1**; but **A0** if **final** answer negative

Q12, (OCR 4721, Jun 2015, Q9)

(i)		$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1 A1	Attempt to differentiate, at least two non-zero terms correct Fully correct
		When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$
		$\frac{dy}{dx} = 0$ gives $a = 13$	M1 A1 **[5]**	Sets their $\frac{dy}{dx}$ to 0. Must be seen
(ii)		$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign
		When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	A1 **[2]**	**www**
(iii)		$6x^2 - 26x + 8 = 0$ $(3x - 1)(x - 4) = 0$ $x = \frac{1}{3}$	M1 M1 A1 **[3]**	Sets their derivative to zero Correct method to solve quadratic **(appx 1)** oe

Q13, (OCR 4721, Jun 2016, Q8)

(i)		$y_1 = 50$, $y_2 = 2(5 + h)^2$ $\frac{(50+20h+2h^2)-50}{(5+h)-5}$ $20 + 2h$	**B1** **M1** **A1** **[3]**	Finds y coordinates at 5 and $5 + h$ Correct method to find gradient of a line segment; at least 3/4 values correct Fully correct working to give answer **AG**
(ii)		e.g. "As h tends to zero, the gradient will be 20"	**B1** **[1]**	Indicates understanding of limit **See Appendix 2 for examples**
(iii)		Gradient of normal $= -\frac{1}{20}$ $y - 50 = -\frac{1}{20}(x - 5), x = 0$ 50¼	**B1** **M1** **A1** **[3]**	 Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\frac{201}{4}, \frac{1005}{20}$

Q14, (OCR 4721, Jun 2016, Q11)

$y = 4x^2 + ax^{-1} + 5$	**B1**	ax^{-1} **soi**
$\frac{dy}{dx} = 8x - ax^{-2}$	**M1**	Attempt to differentiate – at least one non-zero term correct
	A1	Fully correct
At stationary point, $8x - ax^{-2} = 0$	**M1**	Sets their derivative to 0
$a = 8x^3$ **oe**	**A1**	Obtains expression for a in terms of x, or x in terms of a **www**
When $a = 8x^3$, $y = 32$ $32 = 4x^2 + 8x^2 + 5$	**M1**	Substitutes their expression and 32 into equation of the curve to form single variable equation
$x = \frac{3}{2}$ **oe**	**A1**	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.
$a = 27$	**A1** **[8]**	Obtains correct value for a. Ignore –27 given as well.
OR		
$y = 4x^2 + ax^{-1} + 5$	**B1**	ax^{-1} **soi**
$\frac{dy}{dx} = 8x - ax^{-2}$	**M1**	Attempt to differentiate – at least one non-zero term correct
	A1	Fully correct
$32 = 4x^2 + ax^{-1} + 5$	**M1**	Substitutes 32 into equation of the curve to find expression for a
$a = 27x - 4x^3$	**A1**	Obtains expression for a in terms of x **www**
At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$	**M1**	Sets derivate to zero **and** forms single variable equation
$x = \frac{3}{2}$ **oe**	**A1**	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.
$a = 27$	**A1**	Obtains correct value for a. Ignore –27 given as well.

Q15, (OCR 4721, Jun 2017, Q11)

(i)		Gradient of given line = 6	B1	soi as gradient of the line
		Perpendicular gradient = $-\frac{1}{6}$	M1	Uses product of perpendicular gradients is – 1 at some point; may be implied by later working.
		$\frac{dy}{dx} = -2kx^{-3}$	M1 A1	Attempt to differentiate (ax^{-3} seen) Fully correct
		$-\frac{1}{6} = -2k(-3)^{-3}$	M1	Equates their derivative at $x = -3$ with their perpendicular gradient
		$k = -\frac{9}{4}$	A1 **[6]**	Correct value of k. Allow $-\frac{27}{12}$ etc.
(ii)		When $x = -3$, $y = -\frac{9}{4(-3)^2} = -\frac{1}{4}$	B1	Correct value of y www
		$y + \frac{1}{4} = 6(x + 3)$	M1	Attempts equation of straight line through $(-3, y)$, any non-zero gradient. y must be from their k but allow slips for M mark.
			A1ft	Correct equation in any form – gradient 6 but ft their value of $\frac{k}{9}$. Allow $6\,(x - -3)$
		$24x - 4y + 71 = 0$	A1 **[4]**	Correct equation in required form i.e. $a(24x - 4y + 71) = 0$ for integer a, terms in any order. **cao**

Q16, (Jun 2010, Q10)

(i)	$\frac{dy}{dx} = 4x^3$	**M1**	
	when $x = 2$, $\frac{dy}{dx} = 32$ s.o.i.	**A1**	i.s.w.
	when $x = 2$, $y = 16$ s.o.i.	**B1**	
	$y = 32x - 48$ c.a.o.	**A1**	
(ii)	34.481	**2**	**M1** for $\frac{2.1^4 - 2^4}{0.1}$
(iii) (*A*)	$16 + 32h + 24h^2 + 8h^3 + h^4$ c.a.o.	**3**	**B2** for 4 terms correct **B1** for 3 terms correct
(iii) (*B*)	$32 + 24h + 8h^2 + h^3$ or ft	**2**	**B1** if one error
(iii) (*C*)	as $h \to 0$, result $\to$ their 32 from (iii) (B)	**1**	
	gradient of tangent is limit of gradient of chord	**1**	

Q17, (OCR H230/02, Sample Question Paper, Q7)

$f(x+h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$	M1	1.1	Attempt at expansion with product of powers of x and h summing to 4 and some attempt at coefficients, not necessarily correct
$\frac{f(x+h) - f(x)}{h} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$	M1	1.1	Attempt $\frac{f(x+h) - f(x)}{h}$ Allow at most two errors
$= 4x^3 + 6x^2h + 4xh^2 + h^3$	A1	1.1	All terms correct
As $h \to 0$ all the terms in h tend to zero.	A1	2.4	Accept some indication that as h tends to 0, the terms involving h vanish and leave $4x^3$
Therefore $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 4x^3$	E1	2.1	Award for good use of language, and of limit and function notation
	[5]		