Differential Equations (Year 1) (From OCR 4722)

Q1, (Jan 2007, Q3)

(i) $\int(4x-5)dx = 2x^2 - 5x + c$	M1		Obtain at least one correct term
	A1	**2**	Obtain at least $2x^2 - 5x$
(ii) $y = 2x^2 - 5x + c$	B1√		State or imply y = their integral from **(i)**
$7 = 2\times3^2 - 5\times3 + c \Rightarrow c = 4$	M1		Use (3,7) to evaluate c
So equation is $y = 2x^2 - 5x + 4$	A1	**3**	Correct final equation
		5	

Q2, (Jan 2008, Q5)

$\int 12x^{\frac{1}{2}}dx = 8x^{\frac{3}{2}}$	M1		Attempt to integrate
	A1√		Obtain correct, unsimplified, integral following their f(x)
	A1		Obtain $8x^{\frac{3}{2}}$, with or without $+c$
$y = 8x^{\frac{3}{2}} + c \Rightarrow 50 = 8\times4^{\frac{3}{2}} + c$	M1		Use (4, 50) to find c
$\Rightarrow c = -14$	A1√		Obtain c = -14, following $kx^{\frac{3}{2}}$ only
Hence $y = 8x^{\frac{3}{2}} - 14$	A1	**6**	State $y = 8x^{\frac{3}{2}} - 14$ aef, as long as single power of x
		6	

Q3, (Jun 2011, Q2)

(i)	$\int\left(6x^{\frac{1}{2}}-1\right)dx=4x^{\frac{3}{2}}-x+c$	M1		Obtain $kx^{\frac{3}{2}}$	Any k, as long as numerical. Allow both M1 and A1 for equiv eg $x\sqrt{x}$
		A1		Obtain $4x^{\frac{3}{2}}$	Allow for unsimplified coefficient as well (ie $^{6}/_{1.5}$).
		B1	3	Obtain $-x$ (don't penalise lack of $+c$)	Allow $-1x$. Maximum of 2 marks if $\int$ or dx still present in final answer. Maximum of 2 marks if not given as one expression – eg the two terms are integrated separately and never combined.
(ii)	$y=4x^{\frac{3}{2}}-x+c$ $17=32-4+c \Rightarrow c=-11$ hence $y=4x^{\frac{3}{2}}-x-11$	M1*		State or imply $y=$ their integral from (i)	Must have come from integration attempt ie increase in power by 1 for at least one term, but allow if -1 disappeared in part (i) ie at least one of the M1 and the B1 must have been awarded in part (i). Can still get this M1 if no $+c$. The y does not have to be explicit – it could be implied by eg 17 = F(4). M0 if they start with $y=$ their integral from (i), but then attempt to use $y-17=m(x-4)$. This is a re-start and gains no credit.
		M1d*		Attempt to find c using (4, 17)	M0 if no $+c$. M0 if using $x=17$, $y=4$.
		A1	3	Obtain $y=4x^{\frac{3}{2}}-x-11$	Coefficients now need to be simplified, so $-1x$ is A0. Allow A1 for equiv eg $x\sqrt{x}$ Must be an equation ie $y=\ldots$, so A0 for 'equation = …' or 'f(x) = …'
			6		

Q4, (Jun 2012, Q2)

(i)		$\int(x^2-2x+5)\,dx=\frac{1}{3}x^3-x^2+5x+c$	M1	Attempt integration	An increase in power by 1for at least 2 terms. Allow if the +5 disappears.
			A1	Obtain two correct (algebraic) terms	Allow if the coefficient if x^2 isn't yet simplified.
			A1	Obtain fully correct expression (allow no + c)	Allow if the coefficient if x^2 isn't yet simplified. A0 if integral sign or dx still present in their answer (but allow $\int$ = ...). A0 if a list of terms rather than an expression.
			[3]		
(ii)		$y=\frac{1}{3}x^3-x^2+5x+c$	M1*	State or imply y = their integral from (i)	Must have come from integration attempt ie the M1 must have been gained in part (i). Allow slips when transferring expression from (i). Can still get this M1 if no + c. The y does not have to be explicit - it could be implied by eg 11 = F(3) (but not by 3 = F(11)). Using definite integration with limits of 3 & 11 is M0. M0 if they start with y = their integral from (i), but then attempt to use y - 11 = m (x - 3). This is a re-start and gains no credit.
		$11=9-9+15+c\Rightarrow c=-4$	M1d*	Attempt to find c using (3, 11)	Need to get as far as attempting c. M1 could be implied by eg 11 = 9 – 9 + 15 and then an attempt to include a constant to balance the eqn, even though + c never actually seen. M0 if no + c seen or implied. M0 if using x = 11, y = 3.
		hence $y=\frac{1}{3}x^3-x^2+5x-4$	A1	Obtain $y=\frac{1}{3}x^3-x^2+5x-4$	Coeff of x^2 now needs to be simplified (A0 for $-1x^2$). Must be an equation ie y = ..., so A0 for 'f(x) = ...' or 'equation = ...' Allow aef, such as $3y=x^3-3x^2+15x-12$.
			[3]		

Q5, (Jun 2015, Q5)

${}^{dy}/_{dx} = 6x^{0.5} + c$	M1*	Attempt integration	Must be of form $px^{0.5}$, any (non-zero) numerical p, and no other algebraic terms
	A1	Obtain $6x^{0.5}$ (allow no + c)	Allow unsimplified coeff ie ${}^{3}/_{0.5}$, even if subsequently incorrect No need to see ${}^{dy}/_{dx}$ =, and ignore if incorrect (eg y = ...)
$5 = 12 + c$	M1d*	Attempt to use $x = 4$, gradient = 5	Must follow attempt at integration M0 if no $+c$ Condone incorrect notation (eg y = ...) as long as 5 used correctly Attempt to use $x = 4$, ${}^{dy}/_{dx} = 5$ – allow slip as long as intention clear
$c = -7$	A1	Rearrange to obtain $c = -7$	No need to see explicit expression for ${}^{dy}/_{dx}$
$y = 4x^{1.5} - 7x + k$	M1 dd*	Attempt second integration	Must be of form $qx^{1.5} + rx$, any (non-zero) numerical q, r, and no other algebraic terms Dependent on at least M1 M1 awarded
$1 = 32 - 28 + k$, hence $k = -3$	M1 ddd*	Attempt to find k using (4, 1)	Condone notation for the constant of integration being the same as previously used Dependent on all previous M marks Attempt to use $x = 4$, $y = 1$
$y = 4x^{1.5} - 7x - 3$	A1	Obtain $y = 4x^{1.5} - 7x - 3$	Coefficients must now be simplified Must be an equation, ie y = ..., so A0 for 'f(x) = ...' or 'equation = ...'
	[7]		

Q6, (Jun 2018, Q8)

$y = x^2 - \frac{1}{2}ax^{-2} + 3x + c$	M1*	Attempt integration	At least two terms to increase in power by 1
	A1	Obtain correct integral	Condone no $+ c$ Condone no '$y =$' Allow unsimplified coefficients
$8 = 1 - \frac{1}{2}a + 3 + c$	M1	Attempt to find equation in a and c using (1, 8)	Must follow attempt at integration ie two terms increasing in power by 1 M1 could be implied by eg $8 = 1 - \frac{1}{2}a + 3$ followed by an attempt to balance the equation M0 if no $+ c$ seen or implied M0 for $x = 8$, $y = 1$ Allow a slip when substituting, as long as it is clear that use of $x = 1$, $y = 8$ is intended
$c = 4 + \frac{1}{2}a$	A1	Obtain correct equation in c and a	Could be explicit or as part of the correct $y = f(x)$ equation

$\frac{1}{3}x^3 + \frac{1}{2}ax^{-1} + \frac{3}{2}x^2 + 4x + \frac{1}{2}ax$ OR $\frac{1}{3}x^3 + \frac{1}{2}ax^{-1} + \frac{3}{2}x^2 + cx$ OR $\frac{1}{3}x^3 + (c-4)x^{-1} + \frac{3}{2}x^2 + cx$	M1d* A1	Attempt integration Obtain correct integral	Must follow first attempt at integration, and include either c or an attempt at c in terms of a (and possibly even have a in terms of c) At least three terms to increase in power by 1 Any correct integral in terms of a and/or c
$(9 + \frac{1}{6}a + \frac{27}{2} + 12 + \frac{3}{2}a)$ $-(\frac{1}{3} + \frac{1}{2}a + \frac{3}{2} + 4 + \frac{1}{2}a)$	M1**	Attempt correct use of limits and equate to 30	Correct order and subtraction Must be using limits of 1 and 3 Dependent on M1M1 for the two integration attempts
$\frac{86}{3} + \frac{2}{3}a = 30$ $\frac{2}{3}a = \frac{4}{3}$	M1d**	Attempt to solve for either a or c	Solving for c gives $c = 5$ Another valid method is to find an equation involving a and c from use of limits ($6c - a = 28$) and solve simultaneously with their $c = 4 + \frac{1}{2}a$
$a = 2$	A1 **[9]**	Obtain $a = 2$	