Circles Exam Questions MS (from OCR 4721)

Q1, (Jun 2006, Q9)

(i)	$\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
	(7, 2)	A1	2	(7, 2) (integers required)
(ii)	$\sqrt{(7-4)^2+(2--2)^2}$	M1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
	$=\sqrt{3^2+4^2}$ $=5$	A1	2	5
(iii)	$(x-7)^2+(y-2)^2=25$	B1√		$(x-7)^2$ and $(y-2)^2$ used (*their* centre)
		B1√		$r^2 = 25$ used (*their* r^2)
		B1	3	$(x-7)^2+(y-2)^2=25$ cao <u>Expanded form:</u> $-14x$ and $-4y$ used B1√ $r = \sqrt{g^2+f^2-c}$ used B1√ $x^2+y^2-14x-4y+28=0$ B1 cao <u>By using ends of diameter:</u> $(x - 4)(x - 10) + (y + 2)(y - 6) = 0$ Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1
(iv)	Gradient of $AB = \frac{6--2}{10-4}=\frac{4}{3}$	B1		oe
	Gradient of tangent $= -\frac{3}{4}$	B1√		
		M1		Correct equation of straight line through A, any non-zero gradient
	$y--2=-\frac{3}{4}(x-4)$	A1		
	$3x+4y=4$	A1	5	a, b, c need not be integers

Q2, (Jan 2007, Q10)

10 (i)	Centre $(-1, 2)$ $(x+1)^2 - 1 + (y-2)^2 - 4 - 8 = 0$ $(x+1)^2 + (y-2)^2 = 13$ Radius $\sqrt{13}$	B1 M1 A1	 3	Correct centre Attempt at completing the square Correct radius <u>Alternative method:</u> Centre $(-g, -f)$ is $(-1, 2)$ B1 $g^2 + f^2 - c$ M1 Radius $= \sqrt{13}$ A1
(ii)	$(2)^2 + (k-2)^2 = 13$ $(k-2)^2 = 9$ $k - 2 = \pm 3$ $k = -1$	M1 M1 A1	 3	Attempt to substitute x = -3 into circle equation Correct method to solve quadratic $k = -1$ (negative value chosen)
(iii)	EITHER $y = 6 - x$ $(x+1)^2 + (6 - x - 2)^2 = 13$ $(x+1)^2 + (4 - x)^2 = 13$ $x^2 + 2x + 1 + 16 - 8x + x^2 = 13$ $2x^2 - 6x + 4 = 0$ $2(x-1)(x-2) = 0$ $x = 1, 2$ $\therefore y = 5, 4$ OR $x = 6 - y$ $(6 - y + 1)^2 + (y-2)^2 = 13$ $(7 - y)^2 + (y-2)^2 = 13$ $49 - 14y + y^2 + y^2 - 4y + 4 = 13$ $2y^2 - 18y + 40 = 0$ $2(y-4)(y-5) = 0$ $y = 4, 5$ $\therefore x = 2, 1$	M1 M1 A1 M1 A1 A1	 6	Attempt to solve equations simultaneously Substitute into their circle equation for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$ Both x values correct Both y values correct <u>or</u> one correct pair of values **www** B1 second correct pair of values B1 **SR** **<u>T & I</u>** M1 A1 One correct x (or y) value A1 Correct associated coordinate

Q3, (Jan 2010, Q8)

(i)	Centre $(-3, 2)$	**B1**		
	$(x+3)^2-9+(y-2)^2-4-4=0$	**M1**		Correct method to find r^2
	$r^2=17$			
	$r=\sqrt{17}$	**A1**	3	Correct radius
(ii)	$x^2+(3x+4)^2+6x-4(3x+4)-4=0$	**M1***		substitute for x/y or attempt to get an equation in 1 variable only
		A1		correct unsimplified expression
	$10x^2+18x-4=0$	**A1**		obtain correct 3 term quadratic
	$(5x-1)(x+2)=0$	**M1 dep**		correct method to solve their quadratic
	$x=\frac{1}{5}$ or $x=-2$	**A1**		
	$y=\frac{23}{5}$ or $y=-2$	**A1**	6	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
			9	

Q4, (Jan 2011, Q9)

(i)	Centre (4, 1)	B1		Correct centre
	$(x-4)^2+(y-1)^2-16-1-3=0$	M1		Correct method to find r^2
	$(x-4)^2+(y-1)^2=20$			
	Radius $=\sqrt{20}$	A1	3	Correct radius
(ii)	$k=1\pm\sqrt{20}$	M1 A1ft		y ordinate of their centre ± their radius or Both correct, unsimplified values
	$k=1\pm 2\sqrt{5}$	A1	3	cao
(iii)	$MT^2=r^2-2^2$	M1		Correct use of Pythagoras' theorem involving MT (or SM)
	$MT=4$	A1ft		Correct value of MT for their r
	$ST=8$	A1	3	cao
(iv)	$x=2y+12$	M1*		Attempt to solve equations simultaneously
	$(2y+8)^2+(y-1)^2=20$	A1		Correct unsimplified expression, may be $(12+2y)^2+y^2-8(12+2y)-2y-3=0$
	$4y^2+32y+64+y^2-2y+1=20$			
	$5y^2+30y+45=0$	A1		Obtain correct 3 term quadratic
	$y^2+6y+9=0$			
	$(y+3)^2=0$	DM1		Correct method to solve quadratic of form $ax^2+bx+c=0$ $(b\neq 0)$
	$y=-3$	A1		y value correct, no extra solutions
	$x=6$	A1		x value correct **ISW**
	OR			
	$y-1=-2(x-4)$	M1		Attempt to find equation of radius/normal
		A1		Correct equation
	Solve simultaneously with $y=\frac{1}{2}x-6$	M1		
	$x=6$	A1		
	$y=-3$	A1		
	States line is tangent as meets at one point or verifies (6, -3) lies on circle	B1	6 15	Allow showing distance between (6,-3) and (4,1) = √20

Q5, (Jun 2010, Q10)

(i)		Centre (5, –2) Radius = 5 Diameter = 10	B1 M1 A1 **[3]**	 5 or $\sqrt{25}$ soi
(ii)		Gradient of line = $\frac{2-^-2}{7-5}(=2)$ $y-2=2(x-7)$ or $y-^-2=2(x-5)$ $y=2x-12$	M1 A1 M1 A1 **[4]**	uses $\frac{y_2-y_1}{x_2-x_1}$ with their centre correct equation of straight line through (7, 2) or their centre, any non-zero gradient o.e. 3 term equation
(iii)		$\sqrt{(7-5)^2+(2-^-2)^2}$ $=\sqrt{20}$ $\sqrt{20}<5$ so P lies inside the circle	M1 A1 B1 FT **[3]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ with their centre Compares their length CP with their radius and states consistent conclusion. **Both lengths must be mentioned.**
(iv)		$(x-5)^2+(2x+2)^2(=25)$ $(x-5)^2+(2x+2)^2=25$ $x^2-10x+25+4x^2+8x+4=25$ $5x^2-2x+4=0$ $b^2-4ac=4-(4\times5\times4)$ $b^2-4ac<0$ so no real roots	M1* A1 A1 M1dep A1 **[5]**	Substitute for x/y or attempt to eliminate one of the variables Correct unsimplified equation (= 0 can be implied) Obtain correct 3 term quadratic Attempt to determine whether equation has real roots with consistent conclusion regarding roots/intersection Fully justified statement that line and circle do not meet **www**

Q6, (Jun 2014, Q9)

i)		y coordinate of the centre is -5 Radius = 5 Centre is five units below x axis and radius is five, so just touches the x-axis	**B1** **B1** **B1** **[3]**	Correct y value Correct radius Correct explanation based on the above – allow clear diagram **www**
ii)		$CP^2 = (6-2)^2 + (k+5)^2$ $CP^2 < 25 \Rightarrow 16 + k^2 + 10k + 25 < 25$ $k^2 + 10k + 16 < 0$ $(k+2)(k+8)<0$ $-8 < k < -2$	**M1** **A1** **A1** **M1** **A1** **[5]**	Attempt to find CP or CP^2 Correct three term quadratic expression* $k = -2$ and $k = -8$ found Chooses "inside region" for their roots of their quadratic Must be strict inequalities for the A mark * Or $(k+5)^2 < 9$
iii)		$(2y-2)^2 + (y+5)^2 = 25$ $5y^2 + 2y + 4 = 0$ $b^2 - 4ac = 4 - 4 \times 5 \times 4$ $= -76$ < 0 ,so line and circle do not meet	**M1*** **A1** **M1dep*** **A1** **[4]**	Attempts to eliminate x or y from equation of circle Correct three term quadratic obtained Correct method to establish quadratic has no roots e.g. considers value of b^2-4ac, tries to find roots from quadratic formula Correct clear conclusion **www AG**

Q7, (Jun 2015, Q10)

(i)		$C = (5, -2)$ $(x-5)^2 + (y+2)^2 - 25 = 0$ Radius = 5	B1 M1 A1 **[3]**	Correct centre $(x \pm 5)^2 - 5^2$ and $(y \pm 2)^2 - 2^2$ seen (or implied by correct answer) Correct radius – do not allow **A** mark from $(x+5)^2$ and/or $(y-2)^2$
(ii)		Gradient $PC = \frac{2--2}{8-5} = \frac{4}{3}$ Gradient of tangent $= -\frac{3}{4}$ $y - 2 = -\frac{3}{4}(x-8)$ $4y + 3x = 32$	M1 A1 B1ft M1 A1 **[5]**	Attempt to find gradient of radius (3/4 correct) $\frac{-1}{\text{their gradient}}$ processed Equation of straight line through P, using their perpendicular gradient (not from rearrangement) Rearrange to required form **www AG**
(iii)		$Q = (0, -2)$ $R = (0, 8)$ Area $= \frac{1}{2} \times (8 - -2) \times 8$ 40	B1 B1 M1 A1 **[4]**	Q found correctly R found correctly Attempt to find area of triangle with their Q, R and height 8 i.e. $\frac{1}{2} \times (y_R - y_Q) \times 8$

Q8, (Jun 2016, Q10)

(i)		Centre of circle (4, 3) $(x-4)^2-16+(y-3)^2-9-20=0$ $r^2=45$ $r=\sqrt{45}$	**B1** **M1** **A1** **[3]**	Correct centre $(x \pm 4)^2 - 4^2$ and $(y \pm 3)^2 - 3^2$ seen (or implied by correct answer) $\sqrt{45}$ or better **www**
(ii)		At A, $y = 0$ so $x^2 - 8x - 20 = 0$ $(x - 10)(x + 2) = 0$ A = (10, 0) Gradient of radius = $\frac{3-0}{4-10}=-\frac{1}{2}$ Gradient of tangent = 2 $y-0=2(x-10)$ $y=2x-20$	**M1** **A1** **M1** **B1** **M1** **A1** **[6]**	Valid method to find A e.g. put $y = 0$ and attempt to solve quadratic (allow slips) or Pythagoras' theorem Correct answer found Attempts to find gradient of radius (3 out of 4 terms correct for their centre, their A) Equation of line **through their A**, any non-zero gradient Correct answer in any three-term form
(iii)		A' = (–2, 6) $y-6=2(x+2)$ $y=2x+10$	**B1** **M1** **A1** **[3]**	Finds the opposite end of the diameter Line through their A' parallel to their line in (ii) Correct answer in any three-term form
(iv)		OC = $\sqrt{3^2+4^2}=5$ $(0 <)\ r < \sqrt{45} - 5$	**M1** **A1** **[2]**	Attempts to find the distance from O to their centre **and subtract from their radius** Correct inequality, condone $\leq$