

Kinematics with Variable Acceleration MS (From OCR 4728)

Q1, (Jan 2006, Q3)

(i)	$a = 7.2 - 0.9t$ $T = 8$ <u>See also special case in appendix.</u>	M1 A1 M1 A1	[4]	For using $a = dv/dt$ For attempting to solve $a(t) = 0$
(ii)	$v(T) = 28.8$ <u>See also special case in appendix.</u>	B1	[1]	AG (From $7.2 \times 8 - 0.45 \times 8^2$)
(iii)	$s = 3.6t^2 - 0.15t^3 \quad (+C)$ $s = 153.6 (+C)$ s at constant speed = 662.4 Displacement is 816 m	M1 A1 DM1 A1 B1ft A1ft	[6]	For using $s = \int vdt$ For finding $s(T \text{ or } 31)$ or using limits (0) to T or (0) to 31 (dep on integration) Condone $+C$ For using $(31 - cv T) \times 28.8$ $cv 153.6 + cv 662.4$ (non-zero numerical)

Q2, (Jan 2007, Q5)

(i)	$1.8t^2/2 \quad (+C)$ $(t = 0, v = 0) C = 0$ Expression is $1.8t^2/2$	M*1 B1 A1	3	For using $v = \int a dt$ May be awarded in (ii). Accept c written and deleted. also for $1.8t^2 + c$
(ii)	$0.9t^3/3 \quad (+K)$ 0.3×64 $19.2m \quad AG$	M1 A1	4	For using $s = \int v dt$ SR Award B1 for $(s = 0, t = 0) K = 0$ if not already given in (i), or $+K$ included and limits used. For using limits 0 to 4 (or equivalent)
(iii)	$u = 0.9 \times 4^2$ $s = 14.4 \times 3 + \frac{1}{2} \times 7.2 \times 3^2$ $19.2 + 75.6$ Displacement is 94.8m OR $v = \int 7.2 dt$ $t = 0, v = 14.4, c = 14.4$ $s = \int 7.2t + 14.4 dt$ $t = 0, s = 0, k = 0$ $s = 3.6 \times 3^2 + 14.4 \times 3$ $19.2 + 75.6 = 94.8$ Displacement is 94.8m	D* M1 M1 A1 M1 A1 M1 A1 M1 A1	5	For using 'u' = $v(4)$ For using $s = ut + \frac{1}{2} \times 7.2t^2$ with non-zero u ($s = 75.6$) For adding distances for the two distinct stages For finding $v(4)$ Integration and finding non-zero integration constant Nb Using $t=4, v=14.4$ gives $c = -14.4$ $s = \int 7.2t - 14.4 dt$ Integration and finding integration constant. Nb $t=4$ with $s=19.2$ and $v=7.2t-14.4$ gives $k=19.2$ Substituting $t = 3$ (OR 7 into $s = 3.6t^2 - 14.4t + 19.2$) ($s=75.6$) (OR $s = 3.6 \times 7^2 - 14.4 \times 7 + 19.2$) Adding two distinct stages OR $s = 3.6 \times 7^2 - 14.4 \times 7 + 19.2 = 94.8$ final M1A1

Q3, (Jun 2007, Q6)

6(i)	$v = 0.004t^3 - 0.12t^2 + 1.2t$ $v(10) = 4 - 12 + 12 = 4\text{ms}^{-1}$ (AG)	M1 A1 A1 [3]	For differentiating s Condone the inclusion of +c Correct formula for v (no +c) and t=10 stated sufficient
(ii)	$v = 0.8t - 0.04t^2$ (+ C) $8 - 4 + C = 4$ $v = 0.8 \times 20 - 0.04 \times 20^2$ (+ C) $v(20) = 16 - 16 = 0$ (AG)	M1 A1 M1* M1 DA1 [5]	For integrating a Only for using v(10) = 4 to find C Dependant on M1*
(iii)	$S = 0.4t^2 - 0.04t^3/3$ (+K) $s(10) = 10 - 40 + 60 = 30$ $40 - 40/3 + K = 30 \rightarrow K = 10/3$ $S(20) = 160 - 320/3 + 10/3 = 56.7\text{m}$ OR $s(10) = 10 - 40 + 60 = 30$ $S = 0.4t^2 - 0.04t^3/3$ $S(20) - S(10) = 26.6, 26.7$ displacement is 56.7m	M1 A1 B1 M1 A1 B1 [6] B1 M1 A1 M1 A1 B1	For integrating v Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear) For using S(10) = 30 to find K Not if S includes ct term Accept 56.6 to 56.7, Adding 30 subsequently is not isw, hence B0 For integrating v Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear) Using limits of 10 and 20 (limits 0, 10 M0A0B0) For 53.3 - 26.7 or better (Note S(10) = 26.7 is fortuitously correct M0A0B0) Accept 56.6 to 56.7

Q4, (Jan 2011, Q6)

6 i	$x = \int t^2 - 9 dt$ $x = t^3/3 - 9t$ (+c) Finds x(2) Displacement = $15\frac{1}{3}$ m OR $x(2) = [t^3/3 - 9t]_0^2$ Displacement = $15\frac{1}{3}$ m	M1* A1 D*M1 B1 [4] D*M1 B1	Uses integration of v(t) Award if +c omitted Allow + c or c omitted Accept 15.3, 46/3. Must be +ve Uses limits $[\]_0^2$ on integrated x(t) Must be +ve
ii	t=0 s=0 or s=46/3 hence x(0) or c= 0 or 46/3 Solves $t^2 - 9 = 0$ t = (\pm)3 $x(3) = 3^3/3 - 9 \times 3$ (+ 15.3) $x(3) = -18$ (or -2.67) Dist = 18 m	B1* M1* A1 D*M1 M1 D*B1 [6]	Needs explanation, may be seen in part i May be implied Value of t when direction of motion changes Substitutes cv(t) >2 in integrated x(t) Evaluates c - 18 may be implied award if .. Accept 18(.0) [c=0 assumed]
iii	$a = d(t^2 - 9)/dt$ a = 2t 10 = 2t t = 5 $x(5) (= 5^3/3 - 9 \times 5 + 15.3) = 12$ m OR $[t^3/3 - 9t]_2^5 = 12$ m	M1* A1 D*M1 A1 A1 [5] A1	Uses differentiation of v(t)

Q5, (Jun 2014, Q3)

(i)	3 ms^{-1}	B1 [1]	
(ii)	$x = \int (0.6t^2 + 3) dt$ $x = 0.6t^3/3 + 3t (+ c)$ Substitutes 1.5 in expression for x $x(1.5) = 5.175 \text{ m}$	M1* A1 D*M1 A1 [4]	Integrates v Accept with/without $+ c$ Needs integration and 2 terms in t Only without $+c$. Accept 5.17, 5.18
(iii)	$a = d(0.6t^2 + 3)/dt$ $6 = 2 \times 0.6t$ $v(5) = 18 \text{ ms}^{-1}$	M1* D*M1 A1 [3]	Differentiates v Plus attempt to solve $a(t) = 6$

Q6, (Jun 2015, Q6)

(i)	$v = \int 4 + 12t dt$ $v = 4t + 12t^2/2 (+ c)$ $(t = 0, v = 2) c = 2$ and $v(3) = 4 \times 3 + 12 \times 3^2/2 (+ 2)$ $v = 68 \text{ m s}^{-1}$	M1* A1 D*M1 A1 [4]	Integrates acceleration Award without $(+ c)$ Evaluates constant
(ii)	$\int 4t + 6t^2 (+2) dt$ $x = 4t^2/2 + 6t^3/3 + 2t (+ d)$ $x(3) = 4 \times 3^2/2 + 6 \times 3^3/3 (+ 3 \times 2)$ $x = 78 \text{ m}$	M1* A1ft D*M1 A1 [4]	Integrates velocity accept omission of d for all subsequent marks
(iii)	$k = 132$	B1ft [1]	ft $cv(78) + 54$
(iv)	$v = d(k - 2t^3) / dt$ $v = -2 \times 3t^2$ $v(3) = -6 \times 3^2 (= -54)$ $68m - 54m = 2mv$ $v = 7 \text{ m s}^{-1}$	M1* A1 D*M1 M1 A1 [5]	Differentiates displacement Award even if k wrong earlier Substitutes $t = 3$ Conservation of momentum, must have $2m$, $cv(68)$

Q7, (Jun 16, Q7)

i	$A: v = \int 0.18t \, dt$ $v = 0.18/2 t^2 (+c)$ $9 = 0.09t^2$ $t = 10$	M1* A1 D*M1 A1 [4]
ii	$B: v = d(Ut + 0.08t^3) / dt$ $v = U + 0.24t^2$ $9 = U + 0.24 \times 5^2$ $U = 3$ $SB(5) = 3 \times 5 + 0.08 \times 5^3$ $SB(5) = 25 \text{ m}$	M1* D*M1 A1 A1 [4]
iii	$A: x = \int 0.09t^2 \, dt$ $x = 0.09t^3 / 3$ $x(16) = 0.03 \times 16^3$ $x = 122.88 \text{ (may be implied by later work)}$ $122.88 = 25 + 10 \times 9 + (9 + v)(x1) / 2$ $v = 6.76 \text{ m s}^{-1}$ <p><i>OR</i></p> $122.88 - 25 - 10 \times 9 = 9x1 + \frac{1}{2} a x1^2$ $\text{Deceleration} = 2.24 \text{ m s}^{-2}$ $v = 9 - 2.24 \times 1$ $v = 6.76 \text{ m s}^{-1}$	M1* D*M1 A1 M1 A1 [5] M1 A1

$$a = 12 - 6t$$

$$a = 0 \text{ gives } t = 2$$

$$x = \int (2 + 12t - 3t^2) dx$$

$$2t + 6t^2 - t^3 + C$$

$$x = 3 \text{ when } t = 0$$

$$\text{so } 3 = C \text{ and}$$

$$x = 2t + 6t^2 - t^3 + 3$$

$$x(2) = 4 + 24 - 8 + 3 = 23 \text{ m}$$

M1	Differentiation, at least one term correct.
A1	
F1	Follow their a
M1	Integration indefinite or definite, at least one term correct.
A1	Correct. Need not be simplified. Allow as definite integral. Ignore C or limits
M1	Allow $x = \pm 3$ or argue it is \int_0^2 from A then ± 3
A1	Award if seen WWW or $x = 2t + 6t^2 - t^3$ seen with +3 added later.
B1	FT their t and their x if obtained by integration but not if -3 obtained instead of +3. [If 20 m seen WWW for displacement award SC6] [Award SC1 for position if constant acceleration used for displacement and then +3 applied]
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