Poisson Distribution (From OCR 4733)

Q1, (Jan 2006, Q1)

In a study of urban foxes it is found that on average there are 2 foxes in every 3 acres.

(i) Use a Poisson distribution to find the probability that, at a given moment,

(a) in a randomly chosen area of 3 acres there are at least 4 foxes, [2]

(b) in a randomly chosen area of 1 acre there are exactly 2 foxes. [3]

(ii) Explain briefly why a Poisson distribution might not be a suitable model. [2]

Q2, (Jan 2007, Q5i,ii)

On a particular night, the number of shooting stars seen per minute can be modelled by the distribution Po(0.2).

(i) Find the probability that, in a given 6-minute period, fewer than 2 shooting stars are seen. [3]

(ii) Find the probability that, in 20 periods of 6 minutes each, the number of periods in which fewer than 2 shooting stars are seen is exactly 13. [3]

Q3, (Jun 2008, Q6a)

On average I receive 19 e-mails per (8-hour) working day. Assuming that a Poisson distribution is a valid model, find the probability that in one randomly chosen hour I receive either 3 or 4 e-mails. [4]

Q4, (Jan 2009, Q3)

The number of incidents of radio interference per hour experienced by a certain listener is modelled by a random variable with distribution Po(0.42).

(i) Find the probability that the number of incidents of interference in one randomly chosen hour is

(a) 0,

(b) exactly 1.

[3]

(ii) Find the probability that the number of incidents in a randomly chosen 5-hour period is greater than 3. **[3]**

(iii) One hundred hours of listening are monitored and the numbers of 1-hour periods in which 0, 1, 2, … incidents of interference are experienced are noted. A bar chart is drawn to represent the results. Without any further calculations, sketch the shape that you would expect for the bar chart. (There is no need to use an exact numerical scale on the frequency axis.) **[2]**

Q5, (Jan 2009, Q9i,ii)

Buttercups in a meadow are distributed independently of one another and at a constant average incidence of 3 buttercups per square metre.

(i) Find the probability that in 1 square metre there are more than 7 buttercups. **[2]**

(ii) Find the probability that in 4 square metres there are either 13 or 14 buttercups. **[3]**

Q6, (Jun 2010, Q1)

(i) The number of inhabitants of a village who are selected for jury service in the course of a 10-year period is a random variable with the distribution Po(4.2).

(a) Find the probability that in the course of a 10-year period, at least 7 inhabitants are selected for jury service. **[2]**

(b) Find the probability that in 1 year, exactly 2 inhabitants are selected for jury service. **[3]**

(ii) Explain why the number of inhabitants of the village who contract influenza in 1 year can probably not be well modelled by a Poisson distribution. **[2]**

Q7, (Jan 2012, Q8i-iii)

In a certain fluid, bacteria are distributed randomly and occur at a constant average rate of 2.5 in every 10 ml of the fluid.

(i) State a further condition needed for the number of bacteria in a fixed volume of the fluid to be well modelled by a Poisson distribution, explaining what your answer means. **[2]**

Assume now that a Poisson model is appropriate.

(ii) Find the probability that in 10 ml there are at least 5 bacteria. **[2]**

(iii) Find the probability that in 3.7 ml there are exactly 2 bacteria. **[3]**

Q8, (Jun 2012, Q4i-iii)

In a rock, small crystal formations occur at a constant average rate of 3.2 per cubic metre.

(i) State a further assumption needed to model the number of crystal formations in a fixed volume of rock by a Poisson distribution. **[1]**

In the remainder of the question, you should assume that a Poisson model is appropriate.

(ii) Calculate the probability that in one cubic metre of rock there are exactly 5 crystal formations. **[2]**

(iii) Calculate the probability that in 0.74 cubic metres of rock there are at least 3 crystal formations. **[3]**

Q9, (Jun 2013, Q9)

The managers of a car breakdown recovery service are discussing whether the number of breakdowns per day can be modelled by a Poisson distribution. They agree that breakdowns occur randomly. Manager A says, "it must be assumed that breakdowns occur at a constant rate throughout the day".

(i) Give an improved version of Manager A's statement, and explain why the improvement is necessary. **[2]**

(ii) Explain whether you think your improved statement is likely to hold in this context. **[1]**

Assume now that the number B of breakdowns per day can be modelled by the distribution $\text{Po}(\lambda)$.

(iii) Given that $\lambda = 9.0$ and $P(B > B_0) < 0.1$, use tables to find the smallest possible value of B_0, and state the corresponding value of $P(B > B_0)$. **[2]**

(iv) Given that $P(B = 2) = 0.0072$, show that λ satisfies an equation of the form $\lambda = 0.12e^{k\lambda}$, for a value of k to be stated. Evaluate the expression $0.12e^{k\lambda}$ for $\lambda = 8.5$ and $\lambda = 8.6$, giving your answers correct to 4 decimal places. What can be deduced about a possible value of λ? **[5]**

Q10, (Jun 2014, Q4)

A zoologist investigates the number of snakes found in a given region of land. The zoologist intends to use a Poisson distribution to model the number of snakes.

(i) One condition for a Poisson distribution to be valid is that snakes must occur at constant average rate. State another condition needed for a Poisson distribution to be valid. **[1]**

Assume now that the number of snakes found in 1 acre of a region can be modelled by the distribution Po(4).

(ii) Find the probability that, in 1 acre of the region, at least 6 snakes are found. **[2]**

(iii) Find the probability that, in 0.77 acres of the region, the number of snakes found is either 2 or 3. **[4]**

Q11, (Jun 2015, Q2)

A class investigated the number of dead rabbits found along a particular stretch of road.

(i) The class agrees that dead rabbits occur randomly along the road. Explain what this statement means. **[1]**

(ii) State, in this context, an assumption needed for the number of dead rabbits in a fixed length of road to be modelled by a Poisson distribution, and explain what your statement means. **[2]**

Assume now that the number of dead rabbits in a fixed length of road can be well modelled by a Poisson distribution with mean 1 per 600 m of road.

(iii) Use an appropriate formula, showing your working, to find the probability that in a road of length 1650 m there are exactly 3 dead rabbits. **[3]**

Q12, (Jun 2016, Q4)

It is given that $Y \sim \text{Po}(\lambda)$, where $\lambda \neq 0$, and that $P(Y = 4) = P(Y = 5)$. Write down an equation for λ. Hence find the value of λ and the corresponding value of $P(Y = 5)$. **[5]**

Q13, (Jun 2016, Q6i-ii)

The number of cars passing a point on a single-track one-way road during a one-minute period is denoted by X. Cars pass the point at random intervals and the expected value of X is denoted by λ.

(i) State, in the context of the question, two conditions needed for X to be well modelled by a Poisson distribution. **[2]**

(ii) At a quiet time of the day, $\lambda = 6.50$. Assuming that a Poisson distribution is valid, calculate $P(4 \leqslant X < 8)$. **[3]**
