

**Expectation and Variance of Linear Transformations of Discrete Random Variables (From Edexcel 6683)**

**Q1, (Jun 2006, Q4)**

(a)	$E(X) = 3;$ $Var(X) = \frac{25-1}{12} = 2$ <b>**AG**</b> $Var(X) = 1^2 \times \frac{1}{5} + 2^2 \times \frac{1}{5} + 3^2 \times \frac{1}{5} \dots - 3^2 = 11 - 9 = 2$ <b>**AG**</b> Accept $(55/5) - 9$ as minimum evidence.	B1  M1A1  (3)
(b) M1A1 ∫	$E(3X - 2) = 3E(X) - 2 = 7$	(2)
(c)	$Var(4 - 3x) = 3^2 Var(X) = 18$	M1A1 (2)
		<b>Total 7</b>

**Q2, (Jun 2007, Q7)**

(a)	$p + q = 0.45$ $\sum xP(X = x) = 4.5$ $3p + 7q = 1.95$	B1 M1 A1 (3)
(b)	Attempt to solve equations in (a) $q = 0.15$ $p = 0.30$	M1 A1 A1 (3)
(c)	$P(4 < X < 7) = P(5) + P(7)$ $= 0.2 + q = 0.35$	M1 A1 ∫ (2)
(d)	$Var(X) = E(X^2) - [E(X)]^2 = 27.4 - 4.5^2$ $= 7.15$	M1 A1 (2)
(e)	$E(19 - 4X) = 19 - 4 \times 4.5 = 1$	B1 (1)
(f)	$Var(19 - 4X) = 16Var(X)$ $= 16 \times 7.15 = 114.4$	M1 A1 (2)
		<b>Total 13 marks</b>

**Q3, (Jun 2009, Q6)**

(a)	<table border="1"> <tr> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>3a</td> <td>2a</td> <td>a</td> <td>b</td> </tr> </table>	0	1	2	3	3a	2a	a	b		B1	(1)
0	1	2	3									
3a	2a	a	b									
(b)	$3a + 2a + a + b = 1$ $2a + 2a + 3b = 1.6$ $14a = 1.4$ $a = 0.1$ $b = 0.4$	or equivalent, using Sum of probabilities =1 or equivalent, using $E(X)=1.6$ Attempt to solve	M1 M1 M1dep B1 B1	(5)								
(c)	$P(0.5 < x < 3) = P(1) + P(2)$ $= 0.2 + 0.1$ $= 0.3$	3a or their 2a+their a Require $0 < 3a < 1$ to award follow through	M1 A1 ft	(2)								
(d)	$E(3X - 2) = 3E(X) - 2$ $= 3 \times 1.6 - 2$ $= 2.8$		M1 A1 cao	(2)								
(e)	$E(X^2) = 1 \times 0.2 + 4 \times 0.1 + 9 \times 0.4 (= 4.2)$ $\text{Var}(X) = "4.2" - 1.6^2$ $= 1.64$	**given answer**	M1 M1 A1 cso	(3)								
(f)	$\text{Var}(3X - 2) = 9 \text{Var}(X)$ $= 14.76$	awrt 14.8	M1 A1	(2)								
				<b>[15]</b>								

**Q4, (Jun 2013, Q5a-c)**

(a)	$3a + 2b = 0.7$ $a + 2a + 3a + 4b + 5b + 1.8 = 4.2$ <u>or</u> $6a + 9b = 2.4$ $5b = 1$ $b = \underline{0.2}$ $a = \underline{0.1}$	Attempt to solve	M1 M1 M1 B1 B1 cao cao	(5)
(b)	$E(X^2) = 1 \times 0.1 + 2^2 \times 0.1 + 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.2 + 6^2 \times 0.3 (= 20.4)$ (*)		B1cso	(1)
(c)	$[\text{Var}(X) = ] 20.4 - 4.2^2 [= 2.76]$ $\text{Var}(5 - 3X) = 9 \text{Var}(X)$ $= \underline{24.84}$ or $\underline{24.8}$ (allow $\frac{621}{25}$ )		M1 M1 A1 cao	(3)

**Q5, (Jun 2012, Q1)**

<b>(a)</b>	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math></td> <td><math>-1</math></td> <td><math>0</math></td> <td><math>1</math></td> <td><math>2</math></td> </tr> <tr> <td><math>P(X=x)</math></td> <td><math>4k</math></td> <td><math>k</math></td> <td><math>0</math></td> <td><math>k</math></td> </tr> </table>	$x$	$-1$	$0$	$1$	$2$	$P(X=x)$	$4k$	$k$	$0$	$k$		M1
	$x$	$-1$	$0$	$1$	$2$								
$P(X=x)$	$4k$	$k$	$0$	$k$									
$4k + k + (0) + k = 1$	(Allow verify approach)	A1											
	$6k = 1 \Rightarrow k = \frac{1}{6} \quad (*)$		A1 cso (3)										
<b>(b)</b>	$[E(X)] = -4k (+0+0) + 2k \quad \underline{\text{or}} \quad -2k \quad \underline{\text{or}} \quad -1 \times \frac{4}{6} + 2 \times \frac{1}{6}$		M1										
	$= -\frac{1}{3} \quad (\text{or } -0.3\bar{3})$		A1 (2)										
<b>(c)</b>	$[E(X^2)] = (-1)^2 \times 4k + (0+0) + 2^2 k \quad \underline{\text{or}} \quad 4k + 4k \quad \underline{\text{or}} \quad (-1)^2 \times \frac{4}{6} + 2^2 \times \frac{1}{6} \quad (\text{o.e.})$		M1										
	$= \frac{4}{3} \quad (*)$		A1 cso (2)										
<b>(d)</b>	$[\text{Var}(X)] = \frac{4}{3} - \left(-\frac{1}{3}\right)^2 \quad \underline{\text{or}} \quad 8k - 4k^2 = \left[\frac{11}{9}\right]$	$Y = 1 - 3X: \begin{matrix} 4 & 1 & -2 & -5 \\ \text{Prob:} & 4k & k & 0 & k \end{matrix}$ And $E(Y) = 12k$ $E(Y^2) = 90k$ and $\text{Var}(Y) = 90k - 144k^2$	M1										
	$\text{Var}(1-3X) = (-3)^2 \text{Var}(X) \quad \underline{\text{or}} \quad 9\text{Var}(X)$		M1										
	$= 11$		A1 cao (3)										
			<b>[10]</b>										

**Q6, (Jun 2013(R), Q7a-e)**

<b>(a)</b>	$E(S) = 0 + 1 \times 0.2 + 2 \times 0.1 + 4 \times 0.3 + 5 \times 0.2 = [0.2 + 0.2 + 1.2 + 1.0]$		M1
	<b><u>2.6</u></b>		A1 (2)
<b>(b)</b>	$E(S^2) = 0 + 1 \times 0.2 + 2^2 \times 0.1 + 4^2 \times 0.3 + 5^2 \times 0.2 \quad \underline{\text{or}} \quad 0.2 + 0.4 + 4.8 + 5$		M1
	<b><u>10.4</u> (*)</b>		A1 cso (2)
<b>(c)</b>	$\text{Var}(S) = 10.4 - ("2.6")^2$		M1
	<b><u>3.64</u></b> or $\frac{91}{25}$ (o.e.)		A1 (2)
<b>(d)(i)</b>	$5E(S) - 3 = 5 \times "2.6" - 3, \quad = \underline{\mathbf{10}}$		M1, A1
<b>(ii)</b>	$5^2 \text{Var}(S) = 25 \times 3.64, \quad = \underline{\mathbf{91}}$		M1, A1 (4)
<b>(e)</b>	$5S - 3 > S + 3 \Rightarrow 4S > 6 \quad \text{or} \quad S > 1.5, \quad \text{so need } P(S \geq 2)$		M1, A1
	$P(S \geq 2) = \underline{\mathbf{0.6}}$		A1 (3)

**Q7, (Jan 2010, Q5)**

(a)	$k + 4k + 9k = 1$ $14k = 1$ $k = \frac{1}{14} \quad \text{**given**}$	M1	CSO	A1	(2)
(b)	$P(X \geq 2) = 1 - P(X = 1) \quad \text{or} \quad P(X = 2) + P(X = 3)$ $= 1 - k = \frac{13}{14} \quad \text{or} \quad 0.92857\dots$	M1	awrt 0.929	A1	(2)
(c)	$E(X) = 1 \times k + 2 \times k \times 4 + 3 \times k \times 9 \quad \text{or} \quad 36k$ $= \frac{36}{14} = \frac{18}{7} \quad \text{or} \quad 2\frac{4}{7} \quad \text{(or exact equivalent)}$	M1		A1	(2)
(d)	$\text{Var}(X) = 1 \times k + 4 \times k \times 4 + 9 \times k \times 9 - \left(\frac{18}{7}\right)^2$ $\text{Var}(1 - X) = \text{Var}(X)$ $= \frac{19}{49} \quad \text{or} \quad 0.387755\dots$	M1 M1		M1	(4)
			awrt 0.388	A1	(4)
				<b>Total [10]</b>	