Discrete Random Variables (From OCR 4732)

Q1, (Jan 2006, Q3)

In Mr Kendall's cupboard there are 3 tins of baked beans and 2 tins of pineapple. Unfortunately his daughter has removed all the labels for a school project and so the tins are identical in appearance. Mr Kendall wishes to use both tins of pineapple for a fruit salad. He opens tins at random until he has opened the two tins of pineapples.

Let X be the number of tins that Mr Kendall opens.

(i) Show that $P(X = 3) = \frac{1}{5}$. [4]

(ii) The probability distribution of X is given in the table below.

x	2	3	4	5
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

Find $E(X)$ and $Var(X)$. [5]

Q2, (Jun 2007, Q1)

The table shows the probability distribution for a random variable X.

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

Calculate $E(X)$ and $Var(X)$. [5]

Q3, (Jan 2008, Q6)

The probability distribution for a random variable Y is shown in the table.

y	1	2	3
$P(Y = y)$	0.2	0.3	0.5

(i) Calculate $E(Y)$ and $Var(Y)$. [5]

Another random variable, Z, is independent of Y. The probability distribution for Z is shown in the table.

z	1	2	3
$P(Z = z)$	0.1	0.25	0.65

One value of Y and one value of Z are chosen at random. Find the probability that

(ii) $Y + Z = 3$, [3]

(iii) $Y \times Z$ is even. [3]

Q4, (Jan 2009, Q1)

Each time a certain triangular spinner is spun, it lands on one of the numbers 0, 1 and 2 with probabilities as shown in the table.

Number	Probability
0	0.7
1	0.2
2	0.1

The spinner is spun twice. The total of the two numbers on which it lands is denoted by X.

(i) Show that $P(X = 2) = 0.18$. **[3]**

The probability distribution of X is given in the table.

x	0	1	2	3	4
$P(X = x)$	0.49	0.28	0.18	0.04	0.01

(ii) Calculate $E(X)$ and $Var(X)$. **[5]**

Q5, (Jun 2010, Q5)

Each of four cards has a number printed on it as shown.

1 **2** **3** **3**

Two of the cards are chosen at random, without replacement. The random variable X denotes the sum of the numbers on these two cards.

(i) Show that $P(X = 6) = \frac{1}{6}$ and $P(X = 4) = \frac{1}{3}$. **[3]**

(ii) Write down all the possible values of X and find the probability distribution of X. **[4]**

(iii) Find $E(X)$ and $Var(X)$. **[5]**

Q6, (Jan 2011, Q7)

The probability distribution of a discrete random variable, X, is shown below.

x	0	2
$P(X = x)$	a	$1 - a$

(i) Find $E(X)$ in terms of a. **[2]**

(ii) Show that $Var(X) = 4a(1 - a)$. **[3]**

Q7, (Jun 2013, Q3)

The probability distribution of a random variable X is shown.

x	1	3	5	7
$\mathrm{P}(X=x)$	0.4	0.3	0.2	0.1

(i) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

(ii) Three independent values of X, denoted by X_1, X_2 and X_3, are chosen. Given that $X_1 + X_2 + X_3 = 19$, write down all the possible sets of values for X_1, X_2 and X_3 and hence find $\mathrm{P}(X_1 = 7)$. **[2]**

(iii) 11 independent values of X are chosen. Use an appropriate formula to find the probability that exactly 4 of these values are 5s. **[3]**

Q8, (Jun 2014, Q2)

(a) The probability distribution of a random variable W is shown in the table.

w	0	2	4
$\mathrm{P}(W=w)$	0.3	0.4	0.3

Calculate $\mathrm{Var}(W)$. **[3]**

(b) The random variable X has probability distribution given by

$$\mathrm{P}(X=x) = k(x+1) \quad \text{for } x = 1, 2, 3, 4.$$

(i) Show that $k = \frac{1}{14}$. **[1]**

(ii) Calculate $\mathrm{E}(X)$. **[3]**

Q9, (Jun 2015, Q9)

The random variable X has probability distribution given by

$$\mathrm{P}(X=x) = a+bx \qquad \text{for } x = 1, 2 \text{ and } 3,$$

where a and b are constants.

(i) Show that $3a+6b = 1$. **[2]**

(ii) Given that $\mathrm{E}(X) = \frac{5}{3}$, find a and b. **[4]**

Q10, (Jan 2013, Q1)

When a four-sided spinner is spun, the number on which it lands is denoted by X, where X is a random variable taking values 2, 4, 6 and 8. The spinner is biased so that $\mathrm{P}(X=x) = kx$, where k is a constant.

(i) Show that $\mathrm{P}(X=6) = \frac{3}{10}$. **[2]**

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

Q11, (Jun 2016, Q1)

The table shows the probability distribution of a random variable X.

x	1	2	3	4
$P(X=x)$	0.1	0.3	0.4	0.2

(i) Find $E(X)$ and $Var(X)$. **[5]**

(ii) Three values of X are chosen at random. Find the probability that X takes the value 2 at least twice. **[3]**

Q12, (Jun 2008, Q4)

At a fairground stall, on each turn a player receives prize money with the following probabilities.

Prize money	£0.00	£0.50	£5.00
Probability	$\frac{17}{20}$	$\frac{1}{10}$	$\frac{1}{20}$

(i) Find the probability that a player who has two turns will receive a total of £5.50 in prize money. [3]

(ii) The stall-holder wishes to make a profit of 20p per turn on average. Calculate the amount the stall-holder should charge for each turn. [4]

Q13, (Jun 2012, Q6)

A six-sided die is biased so that the probability of scoring 6 is 0.1 and the probabilities of scoring 1, 2, 3, 4, and 5 are all equal. In a game at a fête, contestants pay £3 to roll this die. If the score is 6 they receive £10 back. If the score is 5 they receive £5 back. Otherwise they receive no money back. Find the organiser's expected profit for 100 rolls of the die. **[5]**