Discrete Random Variables (From OCR 4732)

Q1, (Jan 2006, Q3)

(i)	$^3/_5 \times {}^2/_4 \times {}^1/_3$ or $^2/_5 \times {}^3/_4 \times {}^1/_3$	M1		or $^1/_{10}$ from tree
	x 2 or +	M1		add 2 equal products of 3 probs
	$^3/_5 \times {}^2/_4 \times {}^1/_3 + {}^2/_5 \times {}^3/_4 \times {}^1/_3$	M1		all correct
	$= {}^1/_5$ AG	A1	**4**	Must see correct working
				NB incorrect methods eg $^3/_5 x^2/_4 x^2/_3$
(ii)	Σxp	M1		$\geq$ 3 terms added. Allow arith errors.
	$= 4$	A1		
	$\Sigma x^2 p$ (= 17)	M1		$\geq$ 3 terms added. Allow arith errors
	$- \mu^2$	M1		Indep if +ve result
				$\Sigma(x-\mu)^2 p$ M2; 3 terms: M1 dep +ve result
	$= 1$	A1	**5**	
				Σxp & $\Sigma x^2 p$, if ÷ eg 4: M0A0 ($-\mu^2$ poss M1)
Total		**9**		

Q2, (Jun 2007, Q1)

$(0\times0.1) + 1\times0.2 + 2\times0.3 + 3\times0.4$	M1	$\geq$ 2 non-zero terms correct eg ÷ 4: M0
$= 2(.0)$	A1	
$(0^2\times0.1) + 1\times0.2 + 2^2\times0.3 + 3^2\times0.4$ (= 5)	M1	$\geq$ 2 non-zero terms correct ÷ 4: M0
$- 2^2$	M1	Indep, ft their μ. Dep +ve result
$= 1$	A1	
	5	$(-2)^2\times0.1+(-1)^2\times0.2+0^2\times0.3+1^2\times0.4$:M2
		$\geq$ 2 non-0 correct: M1 ÷ 4: M0
	5	

Q3, (Jan 2008, Q6)

i	Σyp	M1		$\geq$ 2 terms added ÷ 3 or ÷ 6 etc: M0
	$= 2.3$	A1		
	$\Sigma y^2 p$ (= 5.9)	M1		$\geq$ 2 terms added ÷ 3 or ÷ 6 etc: M0
	$- (\Sigma yp)^2$	M1		dep +ve result
	$= 0.61$ oe	A1	5	
				$(-1.3)^2\times0.2+(-0.3)^2\times0.3+0.7^2\times0.5$: M2
				one term correct: M1
				Use of Z: MR, lose last A1 (2.55, 0.4475)
ii	0.2x0.25 + 0.3x0.1 or 0.05 + 0.03 alone	M2		M1 for one product eg correct×2: M1
				or clearly ident (1,2), (2,1): M1
	= 0.08 oe	A1	3	
iii	0.3×0.1 + 0.3×0.25 + 0.3×0.65			
	+ 0.25×0.2 + 0.25×0.5 alone			M1 : any 3, 4 of these prods alone
	or 0.03 + 0.075 + 0.195 + 0.05 + 0.125	M2		or these 5 prods plus 1 extra or repeat
				or (ii) + prod
				or 0.3 + prod or 0.25 + prod
				or clearly identify
				(1,2) (3,2) (2,2) (2,1) (2,3)
	= 0.475 or $^{19}/_{40}$ oe	A1	3	
				M2 for 0.3 + (0.2 + 0.5) × 0.25
				or 0.25 + (0.1 + 0.65) × 0.3
				or 0.3 + 0.25 – 0.3 × 0.25
				or 1 – (0.2+ 0.5)(0.1+0.65)
				M1 for (0.2+ 0.5)(0.1+0.65)
Total		**11**		

Q4, (Jan 2009, Q1)

(i)	$0.2^2 + 0.7 \times 0.1 \times 2$	M2	0.2^2 or 0.7×0.1: M1
	$= 0.18$ **AG**	A1 3	no errors seen NB $2 \times 0.9 \times 0.1 = 0.18$ M0A0
(ii)	$0.28 + 2\times0.18 + 3\times0.04 + 4\times0.01$	M1	$\geq$ 2 terms correct (excl 0×0.49) ÷ 5 (or 4 or 10 etc): M0
	$= 0.8$ oe	A1	
	$0.28 + 2^2\times0.18 + 3^2\times0.04 + 4^2\times0.01$	M1	$\geq$ 2 terms correct (excl $0^2\times0.49$)
	- "0.8"^{2}	M1	dep +ve result
	$= 0.88$ oe	A1 5	cao
			$\Sigma(x-\mu)^2$: 2 terms: M1; 5 terms M2
			$0.8^2\times0.49+0.2^2\times0.28+1.2^2\times0.18+2.2^2\times0.04+3.2^2\times0.01$
			SC Use original table, 0.4:B1 0.44: B1

Q5, (Jun 2010, Q5)

i	$\frac{1}{2}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{1}{3}$ or $\frac{1}{{}^4C_2}$ or $\frac{2}{12}$ ($=\frac{1}{6}$ **AG**)	B1	or 1 out of 6 or 2 out of 12 or $\frac{2!}{4!}\times2$
	$\frac{1}{4}\times\frac{2}{3}$ or $2\times\frac{1}{4}\times\frac{1}{3}$ or $\frac{1}{2}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{1}{3}$	B1	or $\frac{2}{12}$ or $\frac{1}{6}$ or $\frac{1}{3!}$ or $\frac{1}{{}^4C_2}$ or $\frac{2!}{4!}\times2$
	Add two of these or double one ($=\frac{1}{3}$ **AG**)	B1 3	
			or $\frac{2}{{}^4C_2}$ or $4\times\frac{1}{4}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{2}{3}$ or $\frac{4}{12}$ or $\frac{2!}{4!}\times4$ B1B1
			or $\frac{2}{6}$ or $2\times\frac{1}{6}$ or $\frac{2}{3!}$ or $\frac{2!}{3!}$ B1B1
ii	$X = 3, 4, 5, 6$ only, stated or used	B1	Allow repetitions Allow other values with zero probabilities.
	P(X=5) wking as for P(X=4) above or $1 - (\text{"}\frac{1}{6}\text{"} + \frac{1}{3} + \frac{1}{6})$ or $\frac{1}{3}$	M1	
	P(X=3) wking as for P(X=6) above or $1 - (\frac{1}{3} + \text{"}\frac{1}{3}\text{"} + \frac{1}{6})$ or $\frac{1}{6}$	M1	or M1 for total of their probs = 1, dep B1
	3 4 5 6 $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{6}$ oe	A1 4	or P(X=3)=$\frac{1}{6}$, P(X=4)=$\frac{1}{3}$, P(X=5)=$\frac{1}{3}$, P(X=6)=$\frac{1}{6}$ Complete list of values linked to probs
iii	Σxp $= 4\frac{1}{2}$	M1 A1	$\geq$ 2 terms correct ft
	Σx^2p ($=21\frac{1}{6}$) $-\text{'}4\frac{1}{2}\text{'}^2$	M1 M1	$\geq$ 2 terms correct ft Independent except dependent on +ve result
	$= \frac{11}{12}$ or 0.917 (3 sf)	A1 5	
Total		**12**	

Q6, (Jan 2011, Q7)

i	$(0\times a) + 2\times(1-a)$ $= 2 - 2a$ or $2(1-a)$ oe	M1 A1 2	or $2(1-a)$ Not ISW	Condone $2\times 1-a$ NB $2\times(1-a)\div 2$: M0A0 Eg E(X) = $2-2a$; $2-2a=1$; $a=0.5$: M1A0
ii	$(0\times a) + 2^2\times(1-a)$ $-$ "$(2-2a)$"^{2} $= 4 - 4a - 4 + 8a - 4a^2$ $= 4a - 4a^2$ $(= 4a(1-a))$ **AG** $\begin{array}{\|c\|c\|}\hline -2+2a & 2a \\ \hline a & 1-a \\ \hline\end{array}$ M1 $\text{Var}(X) = a(-2+2a)^2 + 4a^2(1-a)$ M1 $4a^3 - 8a^2 + 4a + 4a^2 - 4a^3$ $4a - 4a^2$ A1	M1 M1 A1 3	or $4-4a$ oe $-$ (i)^2 dep contains a; ISW; Indep mk or $4(1-a) - 4(1-a)^2$ $4(1-a)(1-(1-a))$ Correct table oe	Condone $2^2\times 1-a$ $4-4a-4\pm 8a\pm 4a^2$ or $4-4a-4\pm 4a^2$ or equiv M1M1A0 $4-4a-2(1-a)^2$ M1M1A0 Must see this line, correctly obtained Careful: $4-4a-(2-2a)^2 = 4-4a-(4-4a^2) = -4a+4a^2 = 4a(1-a)$ M1M1A0 only
Total		**5**		

Q7, (Jun 2013, Q3)

(i)	$1\times0.4 + 3\times0.3 + 5\times0.2 + 7\times0.1$ $= 3$ $1^2\times0.4 + 3^2\times0.3 + 5^2\times0.2 + 7^2\times0.1$ $-$ "3"^{2} $= 4$	M1 A1 M1 M1 A1 **[5]**	$\geq$ 3 terms correct ÷ eg 4 M0 $\geq$ 3 terms correct ÷ eg 4 M0 Dep +ve result	Use of $\Sigma(x-\bar{x})^2\times p$: $2^2\times0.4 + 0 + 2^2\times0.2 + 4^2\times0.1$ M2 or 2 correct non–zero terms M1
(ii)	775, 757, 577 $\frac{2}{3}$ or 0.667 (3 sf)	B1 B1 **[2]**	Must show all three	Allow repeats, eg list of 6 orders Alt method X_1: 5 or 7, X_2: 5 or 7; X_3: 5 or 7 or X_1, X_2, X_3 can be 5 or 7 B1
(iii)	Binomial stated, or seen or implied with any n & p $^{11}C_4 \times 0.8^7 \times 0.2^4$ $= 0.111$ (3 sf)	B1 M1 A1 **[3]**	eg by $0.8^r \times 0.2^s$ (r,s>1) not just by ^nC_r Correct method Correct answer, no working M1M1A1	NB 0.0388 scores B1M0A0 as it is $^{11}C_5\times0.8^6\times0.8^5$

Q8, (Jun 2014, Q2)

Q9, (Jun 2015, Q9)

(a)		$(0^2 \times 0.3) + 2^2 \times 0.4 + 4^2 \times 0.3$	M1	last two terms correct. NOT eg ÷ 6 or ÷ 3	$2^2 \times 0.3 + (0) + 2^2 \times 0.3$ M2 1st or 3rd term correct M1
		-2^2 or -4	M1	allow $-$ (any number)^2, dep +ve result	
		$= 2.4$	A1		÷ 3 M0M0A0
			[3]		
(b)	**(i)**	$2k + 3k + 4k + 5k = 1$ oe	B1	or $14k = 1$ oe "= 1" is essential	NOT just $2 + 3 + 4 + 5 = 14$ so $k = \frac{1}{14}$
		$(k = \frac{1}{14}$ **AG)**			Allow verification, eg stating that $\frac{2}{14} + \frac{3}{14} + \frac{4}{14} + \frac{5}{14} = 1$
			[1]		
(b)	(ii)	$\frac{2}{14}, \frac{3}{14}, \frac{4}{14}, \frac{5}{14}$ or $\frac{2}{14}, \frac{6}{14}, \frac{12}{14}, \frac{20}{14}$	B1	≥ 3 correct	$2k, 6k, 12k, 20k$ B1
		Σxp	M1	≥ 3 correct terms added	$2k + 6k + 12k + 20k$ or $40k$ M1 ÷ 4 M0A0
		$= \frac{20}{7}$ or $2\frac{6}{7}$ or 2.86 (3 sf) oe, eg $\frac{40}{14}$	A1	SC $1 \times \frac{1}{14} + 2 \times \frac{2}{14} + 3 \times \frac{3}{14} + 4 \times \frac{4}{14}$ (=2.143) B0M1A0	
			[3]		

Q10, (Jan 2013, Q1)

(i)		$2k + 4k + 6k + 8k = 1$ $k = \frac{1}{20}$ AND $6 \times \frac{1}{20} = \frac{3}{10}$ **AG**	M1 A1 **[2]**	or $2 + 4 + 6 + 8 = 20$ M1 Must see both for A1 or $2k + 4k + 6k + 8k = 20k$ M1 $P(X = 6) = \frac{6k}{20k} = \frac{3}{10}$ A1	Must see correct wk'g for $k = \frac{1}{20}$, otherwise M0A0 NB $k \times 6 = \frac{3}{10} \Rightarrow k = \frac{1}{20}$ M0A0 (even if tested by showing that $k = \frac{1}{20}$ gives $\Sigma p = 1$) Just showing $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 1$ M0A0
(ii)		$2 \times \frac{1}{10} + 4 \times \frac{2}{10} + 6 \times \frac{3}{10} + 8 \times \frac{4}{10}$ oe $= 6$ $2^2 \times \frac{1}{10} + 4^2 \times \frac{2}{10} + 6^2 \times \frac{3}{10} + 8^2 \times \frac{4}{10}$ oe (= 10) $-$ '6'^{2} $= 4$	M1 A1 M1 M1 A1 **[5]**	$\geq$ 3 terms correct ft their values of p, dep $\Sigma p = 1$ cao $\geq$ 3 terms correct; ft their values of p; dep $\Sigma p = 1$ ft their values of p; dep +ve result & $\Sigma p = 1$ cao	Allow i.t.o. k for M1 $\div 4$ M0 Allow ito k for M1M1 $\div 4$ M0 NOT $-$ m$^2 \div 4$ $\sqrt{4} = 2$ lose final A1, not ISW, unless labelled sd

Q11, (Jun 2016, Q1)

i		Σxp = 2.7 oe	M1 A1	≥ 3 correct terms	÷ 4 or ÷ 10 etc M0A0
		Σx^2p (= 8.1) - "2.7"^{2} = 0.81 oe	M1 M1 A1 **[5]**	≥ 3 correct terms dep +ve result	$(x$ - '2.7') ≥ 3 correct terms M1 $\Sigma(x$ - '2.7')^2p ≥ 3 correct terms M1 ÷ 4 or ÷ 10 etc M0M0 A0
ii		$0.3^2 \times 0.7 \times 3 + 0.3^3$ or 0.189 + 0.027 oe = $\frac{27}{125}$ or 0.216	M2 A1 **[3]**	or $0.3^2 \times 0.7 + 0.3^3$ M1 or $0.3^2 \times 0.7 \times 3$ or 0.189 oe M1 0.3^2 or 0.09: M0 unless clearly part of correct method $0.3^2\times0.1\times3+0.3^2\times0.4\times3+0.3^2\times0.2\times3 + 0.3^3$ M2 "×3" omitted 3 times or ≥ 2 terms correct M1	1 - ($^3C_1 \times 0.3 \times 0.7^2 + 0.7^3$) M2 or 1 - $(0.3 \times 0.7^2 + 0.7^3)$ M1 SC: M1 for $0.3^2\times0.6\times3+0.3^3$ or 1 - ($^3C_1 \times 0.3 \times 0.6^2 + 0.6^3$) $\times^3C_1$or $\times^3C_2$ instead of ×3 is OK throughout
			8		

Q12, (Jun 2008, Q4)

(i)	$^1/_{20}$ x $^1/_{10}$ or $^1/_{200}$ or 0.005 x 2 = $^1/_{100}$ or 0.01	M1 M1dep A1 3	
(ii)	$E(X) = 0+50x^1/_{10}+500x^1/_{20}$ or $0+0.5x^1/_{10}+5x^1/_{20}$ = 30p = £0.30 or $^3/_{10}$ Charge "30p" + 20p or 0.3 + 0.2 = 50p or 0.50 or 0.5	M1 A1 M1 A1 4	or eg 20 goes: 2 × £0.50 + £5.00 = £6.00 ("£6.00" + 20 × £0.20) ÷ 20 condone muddled units eg 0.3 + 20 x = 20, 70, 520 : M1A1 $20\times^{17}/_{20} + 70\times^1/_{10} + 520\times^1/_{20}$: M1 = 50 A1 x, $(x - 50)$, $(x - 500)$: M1A1 $x\times^{17}/_{20}+(x-50)\times^1/_{10}+(x-500)\times^1/_{20} = 20$: M1 x = 50 : A1 Ignore "£" or "p"
Total		**7**	

Q13, (Jun 2012, Q6)

$(1 - 0.1) \div 5$ (= 0.18)	M1	can be implied, eg by 18	or, using exp no. of 5's & 6's 18×5 or 10×10
3×0.18 or 2×0.18 or 7×0.1 (or result of these)(poss × 100) (3 × 0.18 only scores if using £3, not score of 3. Similarly for 2 × 0.18) .	M1	5×0.18 or 10×0.1 (or result of these)(poss × 100)	
$4 \times 3 \times 0.18$ AND $2 \times 0.18 + 7\times0.1$ (poss × 100) (or 2.16 AND 1.06 or 216 AND 106)	M1	3 AND $5 \times 0.18+10 \times 0.1$ (poss × 100) (or 3 AND 1.9 or 300 AND 190)	300 AND $18 \times 5 + 10 \times 10$ (NB $300+ 100\times0.18 +100\times0.1$ is insuff)
'2.16' – '1.06' or '216' – '106' must be attempt gain on 1,2,3,4 – loss on 5,6	M1 dep any M1	3 – '1.9' or 300 – '190' must be attempt receipt – payout on 5,6	Eg: $300–100\times(5\times0.18+\mathbf{6}\times0.1)=150$ M1M1M0M1A0
E(profit for 100 rolls) = (£)110	A1	E(profit for 100 rolls) = (£)110	Mark one method only Must be matched pair eg 300–106 or 216–190: M1M1M0M0A0
	[5]	NB $300–(0.1\times300+0.18\times300) = 300–84 =216$ M1M1M0M0A0	