## Goodness of Fit Tests (Year 1) (From OCR 4768)

**Q1, (Jun 2010, Q3b)**

Benford's Law predicts the following probability distribution for the first significant digit in some large data sets.

| Digit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Probability | 0.301 | 0.176 | 0.125 | 0.097 | 0.079 | 0.067 | 0.058 | 0.051 | 0.046 |

On one particular day, the first significant digits of the stock market prices of the shares of a random sample of 200 companies gave the following results.

| Digit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency | 55 | 34 | 27 | 16 | 15 | 17 | 12 | 15 | 9 |

Test at the 10% level of significance whether Benford's Law provides a reasonable model in the context of share prices. **[7]**

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**Q2, (Jan 2012, Q3b)**

A medical researcher is looking into the delay, in years, between first and second myocardial infarctions (heart attacks). The following table shows the results for a random sample of 225 patients.

| Delay (years) | 0 – | 1 – | 2 – | 3 – | 4 – 10 |
|---|---|---|---|---|---|
| Number of patients | 160 | 40 | 13 | 9 | 3 |

The mean of this sample is used to construct a model which gives the following expected frequencies.

| Delay (years) | 0 – | 1 – | 2 – | 3 – | 4 – 10 |
|---|---|---|---|---|---|
| Number of patients | 142.23 | 52.32 | 19.25 | 7.08 | 4.12 |

Carry out a test, using a 2.5% level of significance, of the goodness of fit of the model to the data. **[9]**

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**Q3, (Jun 2012, Q4i)**

The numbers of call-outs per day received by a fire station for a random sample of 255 weekdays were recorded as follows.

| Number of call-outs | 0 | 1 | 2 | 3 | 4 | 5 or more |
|---|---|---|---|---|---|---|
| Frequency (days) | 145 | 79 | 22 | 6 | 3 | 0 |

The mean number of call-outs per day for these data is 0.6. A Poisson model, using this sample mean of 0.6, is fitted to the data, and gives the following expected frequencies (correct to 3 decimal places).

| Number of call-outs | 0 | 1 | 2 | 3 | 4 | 5 or more |
|---|---|---|---|---|---|---|
| Expected frequency | 139.947 | 83.968 | 25.190 | 5.038 | 0.756 | 0.101 |

**(i)** Using a 5% significance level, carry out a test to examine the goodness of fit of the model to the data. **[9]**

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**Q4, (Jan 2013, Q4b)**

A calculator has a built-in random number function which can be used to generate a list of random digits. If it functions correctly then each digit is equally likely to be generated. When it was used to generate 100 random digits, the frequencies of the digits were as follows.

| Digit | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 6 | 8 | 11 | 14 | 12 | 9 | 15 | 5 | 14 | 6 |

Use a goodness of fit test, with a significance level of 10%, to investigate whether the random number function is generating digits with equal probability. **[8]**

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**Q5, (Jun 2016, Q2a)**

A genetic model involving body colour and eye colour of fruit flies predicts that offspring will consist of four phenotypes in the ratio 9 : 3 : 3 : 1.

A random sample of 200 such offspring is taken. Their phenotypes are found to be as follows.

| Phenotype | Brown body Red eye | Brown body Brown eye | Black body Red eye | Black body Brown eye |
|---|---|---|---|---|
| Frequency | 125 | 37 | 32 | 6 |
| Relative proportion from model | 9 | 3 | 3 | 1 |

Carry out a test, using a 2.5% level of significance, of the goodness of fit of the genetic model to these data. **[9]**

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**Q6, (Jun 2017, Q2ii)**

In a different round of the contest, the judges were instructed to award only integer marks between 3 and 10 inclusive. One of the organisers believes that the eight possible marks are equally likely to be awarded. To check this he obtains the following random sample of 80 marks awarded.

| Mark | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 5 | 6 | 10 | 9 | 14 | 16 | 14 | 6 |

Carry out a goodness of fit test, with a significance level of 10%, to investigate the organiser's belief. **[8]**

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