Vectors Exam Questions (From OCR 4724)

Q1, (Jun 2005, Q3)

(i)	M1		For (either point) + t(diff betw vectors)
r = (2**i**-3**j**+**k** or –**i**–2**j**–4**k**) + t(3**i**–**j**+5**k**)	A1	**2**	Completely correct including **r** =. AEF
(ii) *L*(2) (**r**) = 3**i**+2**j**–9**k**+s(4**i** – 4**j** + 5**k**)	M1		For point + (s or t) direction vector
L(1)&L(2) must be of form **r** = **a** + t**b**			
2+3t=3+4s, –3–t=2–4s,1+5t= – 9+5s or suitable equivalences	M1		For 2/3 eqns with 2 different parameters
(t,s) = (+/-3,2) or (–/+1,1) or (–/+9,-7)	M1		For solving any relevant pair of eqns
or (+/–4,2) or (0,1) or (–/+8,-7)	A1		For both parameters correct
Basic check other eqn & interp √	B1	**5**	**7**

Q2, (Jun 2008, Q6)

(i) Produce at least 2 of the 3 relevant equations in t and s **M1** $1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$

Solve for t and s **M1**

$(t, s) = (4, -3)$ AEF ***A1**

Subst (4, –3) into suitable equation(s) & show consistency dep***A1** Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for

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(ii) Method for finding magnitude of any vector ***M1** Expect $\sqrt{29}$ and $\sqrt{21}$

Method for finding scalar product of any 2 vectors ***M1** Expect -18

Using $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ AEF for the correct 2 vectors dep***M1** Should be $-\dfrac{18}{\sqrt{29}\sqrt{21}}$

137 (136.8359) or 43.2(43.164…) **A1** 2.39 (2.388236..) or 0.753(0.75335…) rads

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Q3, (Jan 2010, Q2)

(i) Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA})$, $(\overrightarrow{BC}$ or $\overrightarrow{CB})$, $(\overrightarrow{AC}$ or $\overrightarrow{CA})$ M1 irrespect of label; any notation

Use correct method to find scal prod of any 2 vectors M1 or use corr meth for modulus

Use $\overrightarrow{AB}.\overrightarrow{BC} = 0$ or $\dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{|AB||BC|} = 0$ M1 or use $|\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AC}|^2$

Obtain $p = 1$ (dep 3 @ M1) A1 **4**

(ii) Use equal ratios of appropriate vectors M1 or scalar product method

Obtain $p = -8$ A1 **2**

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Q4, (Jan 2010, Q9)

(i) P is $\begin{pmatrix}3\\1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$ B1

direction vector of ℓ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and of $\overrightarrow{OP}$ is their P √B1

Use $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ for $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and their OP M1

$\theta = 35.3$ or better (0.615… rad) A1 **4**

(ii) Use $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}=0$ M1

$1(3+t)-1(1-t)+2(1+2t)=0$ A1

$t=-\dfrac{2}{3}$ A1

Subst. into $\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}$ to produce $\begin{pmatrix}7/3\\5/3\\-1/3\end{pmatrix}$ ISW A1 **4**

(iii) Use $\sqrt{x^2+y^2+z^2}$ where $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is part **(ii)** answer M1

Obtain $\sqrt{\dfrac{75}{9}}$ AEF, 2.89 or better (2.8867513….) A1 **2**

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Q5, (Jun 2013, Q3)

Set up the 3 relevant equations $1+2\lambda=\mu-1 \qquad -\lambda=5-\mu \qquad 3+5\lambda=2-5\mu$	M1	'M' mark so intention must be clear; minor error(s) only accepted
Attempt to find λ or μ from 2 of the equations & then find μ or λ from any of the 3 equations.	M1	Or find λ, say, from (i)(ii) & then from (ii)(iii) [values shown at next stage] – inconsistency dep*A1 also awarded here
$(\lambda,\mu)=$ (3,8) or $(-2\frac{3}{5},2\frac{2}{5})$ or $(-\frac{11}{15},\frac{8}{15})$ or $(3,-3\frac{1}{5})$ or $(-\frac{11}{15},4\frac{4}{15})$ or $(-2\frac{3}{5},-3\frac{1}{5})$ or $(\frac{1}{5},2\frac{2}{5})$ or $(-8\frac{1}{5},8)$ or $(-4\frac{7}{15},\frac{8}{15})$	A1	Accept equivalent proper/improper fractional values or decimal equivalent values
Demonstrate inconsistency i.e. substitute the correct values into a correct equation (but not the immediate last one used)	M1	e.g. after (3,8), subst in iii & write $3+5\times3 \neq 2-5\times8$ or $3+5\times3=2-5\times8 \therefore$ do not intersect
State "skew"	A1	Dep on 3 @ M1 + A1
(a) Identify direction vectors; (b) state "not identical/same/equal/equiv/multiples" or eval cos(angle) & state $\neq 1(\text{or}-1)$; (c) state "not parallel"	B1	dvs must be identified: $\begin{pmatrix}2\\-1\\5\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ Accept any vector notation.
	[6]	

Q6, (Jun 2012, Q5)

(i)		Use $\cos\theta=\dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert b\rvert}$	M1
		Obtain $\left(\cos\theta=\dfrac{6}{12}\right)\ \theta=60$ or $\dfrac{1}{3}\pi$ or 1.05 or better	A1
			[2]
(ii)		Indicate $\mathbf{a}-\mathbf{b}$ is vector joining ends of $\mathbf{a}$ and $\mathbf{b}$ or equiv $\lvert\mathbf{a}-\mathbf{b}\rvert=\lvert\mathbf{a}\rvert-\lvert\mathbf{b}\rvert$, or anything similar, $\rightarrow$ M0	M1
		Use cosine rule correctly on 3, 4 and included (i) angle	M1
		Obtain $\sqrt{13}$ or 3.61 or better (No ft from wrong θ)	A1
			[3]

Q7, (Jan 2013, Q8)

(i)		$AB = \sqrt{(+/-2)^2 + (+/-2^2 + (+/-4)^2)}$ $AD = \sqrt{(+/-2)^2 + (+/-4)^2 + (+/-2)^2}$	B1 B1 **[2]**	oe oe
(ii)		midpoint is (3, 5, 0) Clear method for finding direction vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{j} - \mathbf{k})$ oe or e.g. $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(-3\mathbf{j} + \mathbf{k})$ cao	B1 M1 A1 **[3]**	Accept any reasonable vector notation. Expect $3\mathbf{j} - \mathbf{k}$ or $-3\mathbf{j} + \mathbf{k}$ "**r =**" is essential. No f.t. for wrong mid-point.
(iii)		substitution of $\lambda = +/-5$ or $\mu = +/-4$	M1 **[1]**	Based on correct answer to (ii)
(iv)		Kite	B1 **[1]**	

Q8, (OCR Y531, Sample Question Paper, Q9)

(i)		$-1\times-2+2\times3+1\times k=0$ $\Rightarrow k=-4$	**M1** **A1** **[2]**	1.1 1.1	Attempt the scalar product and set equal to zero soi
(ii)		Equate x and y coordinates: $3+\lambda=6+2\mu \Rightarrow \lambda-2\mu=3$ $2-\lambda=5+\mu \Rightarrow \lambda+\mu=-3$ $\Rightarrow \mu=-2, \lambda=-1$ Consistent with z coordinates since $7+3\times(-1)=4$ and $2-(-2)=4$ So the point of intersection is $(2, 3, 4)$	**M1** **A1** **E1** **A1** **[4]**	2.1 1.1 1.1 1.1	Use coordinates to find μ and λ. Check consistency with third coordinate
(iii)		The vector product find a mutual perpendicular $\begin{pmatrix}1\\-1\\3\end{pmatrix}\times\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}-2\\-7\\3\end{pmatrix}$ $\begin{pmatrix}1\\a\\b\end{pmatrix}=\lambda\begin{pmatrix}-2\\-7\\3\end{pmatrix}$ $\lambda=-\frac{1}{2}$ $a = 3.5,\ b = -1.5$	**M1** **A1** **M1** **M1** **A1** **[5]**	3.1a 1.1 3.1a 1.1 1.1	Attempt the vector product, by any valid method BC

Q9, (OCR 4727, Jan 2008, Q3)

(i)	line segment OA	B1 B1	**2**	For stating line through O *OR* A For correct description **AEF**
(ii)	$(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b}) = \overrightarrow{AP}\times\overrightarrow{BP}$	B1		For identifying $\mathbf{r}-\mathbf{a}$ with $\overrightarrow{AP}$ and $\mathbf{r}-\mathbf{b}$ with $\overrightarrow{BP}$ Allow direction errors
	$= \lvert AP\rvert\lvert BP\rvert \sin\pi\,.\,\hat{\mathbf{n}} = \mathbf{0}$	B1	**2**	For using × of 2 parallel vectors = **0** *OR* $\sin\pi = 0$ *or* $\sin 0 = 0$ in an appropriate vector expression
(iii)	line through O parallel to AB	B1 B1 B1	**3**	For stating line For stating through O For stating correct direction **SR** For $\overrightarrow{AB}$ or $\overrightarrow{BA}$ allow B1 B0 B1
			7	

Q10, (OCR 4727, Jun 2008, Q5i)

METHOD 1

Lines meet where

$(x =)\ k+2\lambda = k+\mu$
$(y =)\ -1-5\lambda = -4-4\mu$
$(z =)\ 1-3\lambda = -2\mu$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

M1 For attempting to solve any 2 equations

$\Rightarrow\ \lambda = -1,\ \mu = -2$ — A1 For correct values of λ and μ

B1 For attempting a check in 3rd equation *OR* verifying point of intersection is on both lines

$\Rightarrow\ (k-2, 4, 4)$ — A1 **6** For correct point of intersection (allow vector)

SR For finding λ *OR* μ and point of intersection, but check, award up to M1 A1 M1 A0 B0 A1

METHOD 2

$d = \dfrac{\lvert[0,3,1]\,.\,[2,-5,-3]\times[1,-4,-2]\rvert}{\lvert\mathbf{b}\times\mathbf{c}\rvert}$ — For using $\mathbf{a}\,.\,\mathbf{b}\times\mathbf{c}$ with appropriate vectors (division by $\lvert\mathbf{b}\times\mathbf{c}\rvert$ is not essential)

$d = c[0,3,1]\,.\,[-2,1,-3] = 0$ — B1 and showing $d = 0$ correctly

$\Rightarrow$ lines intersect

Lines meet where

$(x =)\ (k+)\ 2\lambda = (k+)\ \mu$
$(y =)\ -1-5\lambda = -4-4\mu$
$(z =)\ 1-3\lambda = -2\mu$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

M1 For attempting to solve any 2 equations

$\Rightarrow\ \lambda = -1,\ \mu = -2$ — A1 For correct value of λ *OR* μ

$\Rightarrow\ (k-2, 4, 4)$ — A1 For correct point of intersection (allow vector)

METHOD 3

e.g. $x-k = \dfrac{2(y+1)}{-5} = \dfrac{y+4}{-4}$ — M1 For solving one pair of simultaneous equations

$\Rightarrow\ y = 4$ — A1 For correct value of x, y or z

$\dfrac{z-1}{-3} = \dfrac{y+1}{-5}$ — M1 For solving for the third variable

$x = k-2$ *OR* $z = 4$ — A1 For correct values of 2 of x, y and z

$x-k = \dfrac{z}{-2}$ checks with $x = k-2,\ z = 4$ — B1 For attempting a check in 3rd equation

$\Rightarrow\ (k-2, 4, 4)$ — A1 For correct point of intersection (allow vector)

Q11, (OCR 4727, Jan 2010, Q1)

METHOD 1		
line segment between l_1 and l_2 $= \pm[4,-3,-9]$	B1	For correct vector
$\mathbf{n} = [1,-1,2]\times[2,3,4] = (\pm)[-2,0,1]$	M1* A1	For finding vector product of direction vectors
distance $= \dfrac{\lvert[4,-3,-9].[-2,0,1]\rvert}{\left(\sqrt{2^2+0^2+1^2}\right)} = \dfrac{17}{\left(\sqrt{5}\right)}$	M1 (*dep)	For using numerator of distance formula
$\neq 0$, so skew	A1 **5**	For correct scalar product and correct conclusion
METHOD 2 lines would intersect where		
$\left.\begin{matrix}1+s=-3+2t\\-2-s=1+3t\\-4+2s=5+4t\end{matrix}\right\} \Rightarrow \begin{cases}s-2t=-4\\s+3t=-3\\2s-4t=9\end{cases}$	B1	For correct parametric form for either line
	M1*	For 3 equations using 2 different parameters
	A1	
	M1 (*dep)	For attempting to solve to show (in)consistency
$\Rightarrow$ contradiction, so skew	A1	For correct conclusion
	5	