Roots of Polynomials Exam Questions (From OCR 4725)

Q1, (Jun 2006, Q3)

One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3\text{i}$.

(i) Write down the other root. [1]

(ii) Find the values of p and q. [4]

Q2, (Jun 2005, Q8)

(a) The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

Q3, (Jun 2007, Q6)

The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(b) Use your answer to part **(ii) (a)** to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

Q4, (Jun 2009, Q4)

The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. **[4]**

Q5, (Jun 2009, Q5)

The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[2]**

Q6, (Jun 2010, Q7)

The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. **[7]**

Q7, (Jan 2011, Q9)

The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

Q8, (Jan 2013, Q9)

(i) Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **[3]**

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **[5]**

Q9, (Jun 2014, Q5)

The cubic equation $2x^3 + 3x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u + 2$ to find a cubic equation in u. **[3]**

(ii) Hence find the value of $\frac{1}{\alpha - 2} + \frac{1}{\beta - 2} + \frac{1}{\gamma - 2}$. **[4]**

Q10, (Jun 2014, Q9)

The roots of the equation $x^3 - kx^2 - 2 = 0$ are α, β and γ, where α is real and β and γ are complex.

(i) Show that $k = \alpha - \frac{2}{\alpha^2}$. **[2]**

(ii) Given that $\beta = u + \mathrm{i}v$, where u and v are real, find u in terms of α. **[4]**

(iii) Find v^2 in terms of α. **[4]**

Q11, (Jun 2015, Q10)

The cubic equation $x^3 + 4x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u. **[3]**

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$. **[7]**

Q12, (Jun 2016, Q10)

(i) Use an algebraic method to find the square roots of the complex number $9 + 40\mathrm{i}$. **[6]**

(ii) Show that $9 + 40\mathrm{i}$ is a root of the quadratic equation $z^2 - 18z + 1681 = 0$. **[1]**

(iii) By using the substitution $z = \frac{1}{u^2}$, find the roots of the equation $1681u^4 - 18u^2 + 1 = 0$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real. **[4]**

Q13, (OCR 2605, Jan 2002, Q1)

The equation $x^4 - 6x^3 - 73x^2 + kx + m = 0$ has two positive roots α, β and two negative roots γ, δ. It is given that $\alpha\beta = \gamma\delta = 4$.

(i) Find the values of the constants k and m. [5]

(ii) Show that $(\alpha + \beta)(\gamma + \delta) = -81$. [4]

(iii) Find the quadratic equation which has roots $\alpha + \beta$ and $\gamma + \delta$. [2]

(iv) Find $\alpha + \beta$ and $\gamma + \delta$. [3]

(v) Show that $\alpha^2 - 3(1+\sqrt{10})\alpha + 4 = 0$, and find similar quadratic equations satisfied by β, γ and δ. [6]

Q14, (OCR 2605, Jan 2004, Q1b)

The equation $x^4 + 4x^2 + 3x - 5 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

Using a substitution, or otherwise, find a quartic equation with integer coefficients which has roots

$$\alpha^2 - 1, \quad \beta^2 - 1, \quad \gamma^2 - 1, \quad \delta^2 - 1.$$ [8]

Q15, (OCR 2605, Jan 2005, Q1a)

The equation $8x^4 + 16x^3 + 1 = 0$ has roots α, β, γ and δ.

Use a suitable substitution to find a quartic equation with integer coefficients which has roots

$$8\alpha^3, 8\beta^3, 8\gamma^3 \text{ and } 8\delta^3.$$ [5]
