

Proof By Induction (Inductive Sequences) Exam Questions

From OCR 4725

Q1, (Jan 2008, Q8)

(i)	$u_2 = 4, u_3 = 9, u_4 = 16$	M1 A1	2	Obtain next terms All terms correct
(ii)	$u_n = n^2$	B1	1	Sensible conjecture made
(iii)		B1 M1 A1 A1	4 7	State that conjecture is true for $n = 1$ or 2 Find u_{n+1} in terms of n Obtain $(n + 1)^2$ Statement of Induction conclusion

Q2, (Jun 2009, Q10)

i)	$u_2 = 7, u_3 = 19$	M1 A1 A1	3	Attempt to find next 2 terms Obtain correct answers Show given result correctly
(ii)	$u_n = 2(3^{n-1}) + 1$	M1 A1	2	Expression involving a power of 3 Obtain correct answer
(iii)	$u_{n+1} = 3(2(3^{n-1}) + 1) - 2$ $u_{n+1} = 2(3^n) + 1$	B1ft M1 A1 A1 B1	5 10	Verify result true when $n = 1$ or $n = 2$ Expression for u_{n+1} using recurrence relation Correct unsimplified answer Correct answer in correct form Statement of induction conclusion

Q3, (Jan 2011, Q3)

B1*	Establish result true for $n = 1$ or 2
M1*	Use given result in recurrence relation in a relevant way
A1*	Obtain $2^n + 1$ correctly
depA1 4	Specific statement of induction conclusion

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Q4, (Jan 2013, Q10)

(i)	$\frac{2}{3}, \frac{2}{5}, \frac{2}{7}$	B1 B1 B1 [3]	B1 x 3, Obtain 3 correct values Justify given answer
(ii)	$\frac{2}{2n-1}$	M1 A1 [2]	Fraction, in terms of n , with correct numerator or denominator Obtain correct answer a.e.f.
(iii)	$\frac{2}{2(n+1)-1}$	B1ft M1 A1 A1 B1 [5]	Verify result true when $n = 1$, for their (ii), or $n = 2, 3$ or 4 Expression for u_{n+1} using recurrence relation in terms of n using their (ii) Correct unsimplified answer Correct answer in correct form Specific statement of induction conclusion, previous 4 marks must be earned, $n=1$ must be verified

Q5, (Jun 2016, Q5)

$$3(2 \times 3^n - 1) + 2$$

B1	Show clearly result true for $n = 1$, accept $(u_1) = 2 \times 3 - 1 = 5$
M1	Substitute for u_n in recurrence relation
A1	Establish correct result for u_{n+1} convincingly
B1	Clear statement of induction conclusion, provided 1 st 3 marks earned
[4]	

From OCR 4755

Q6, (Jan 2008, Q6)

<p>(i)</p> $a_2 = 7 \times 7 - 3 = 46$ $a_3 = 7 \times 46 - 3 = 319$	<p>M1 A1 [2]</p>	<p>Use of inductive definition c.a.o.</p>
<p>(ii)</p> <p>When $n = 1$, $\frac{13 \times 7^0 + 1}{2} = 7$, so true for $n = 1$</p> <p>Assume true for $n = k$</p> $a_k = \frac{13 \times 7^{k-1} + 1}{2}$ $\Rightarrow a_{k+1} = 7 \times \frac{13 \times 7^{k-1} + 1}{2} - 3$ $= \frac{13 \times 7^k + 7}{2} - 3$ $= \frac{13 \times 7^k + 7 - 6}{2}$ $= \frac{13 \times 7^k + 1}{2}$	<p>B1</p> <p>E1</p> <p>M1</p> <p>A1</p>	<p>Correct use of part (i) (may be implied)</p> <p>Assuming true for k</p> <p>Attempt to use $a_{k+1} = 7a_k - 3$</p> <p>Correct simplification</p>
<p>But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$ and so true for all positive integers.</p>	<p>E1</p> <p>E1 [6]</p>	<p>Dependent on A1 and previous E1</p> <p>Dependent on B1 and previous E1</p>

Q7, (Jun 2010, Q6)

<p>6(i)</p> $u_2 = \frac{2}{1+2} = \frac{2}{3}, u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	<p>M1 Use of inductive definition A1 c.a.o. [2]</p>
<p>6(ii)</p> <p>When $n = 1$, $\frac{2}{2 \times 1 - 1} = 2$, so true for $n = 1$</p> <p>Assume $u_k = \frac{2}{2k-1}$</p> $\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1+\frac{2}{2k-1}}$ $= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$ $= \frac{2}{2(k+1)-1}$ <p>But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is also true for $k + 1$. Since it is true for $k = 1$, it is true for all positive integers.</p>	<p>B1 Showing use of $u_n = \frac{2}{2n-1}$ E1 Assuming true for k</p> <p>M1 u_{k+1}</p> <p>A1 Correct simplification</p> <p>E1 Dependent on A1 and previous E1 E1 Dependent on B1 and previous E1 [6]</p>

Q8, (Jan 2011, Q6)

<p>(i) (a)</p> <p>(b)</p>	<p>B1* Vertical line depB1 2 Clearly through (4, 0) B1 Sloping line with +ve slope B1 Through (0, -2) B1ft 3 Half line starting on y-axis 45° shown convincingly</p>
<p>(ii)</p>	<p>B1ft Shaded to left of their (i) (a) B1ft Shaded below their (i) (b) must be +ve slope B1ft 3 Shaded above horizontal through their (0, -2) NB These 3 marks are independent, but 3/3 only for fully correct answer.</p>

Q9, (Jun 2012, Q6)

(i)	$a_2 = 3 \times 2 = 6, a_3 = 3 \times 7 = 21$	B1 [1]	cao
(ii)	<p>When $n = 1$, $\frac{5 \times 3^0 - 3}{2} = 1$, so true for $n = 1$</p> <p>Assume $a_k = \frac{5 \times 3^{k-1} - 3}{2}$</p> $\Rightarrow a_{k+1} = 3 \left(\frac{5 \times 3^{k-1} - 3}{2} + 1 \right)$ $= \frac{5 \times 3^k - 9}{2} + 3 = \frac{5 \times 3^k - 9 + 6}{2}$ $= \frac{5 \times 3^k - 3}{2} = \frac{5 \times 3^{(k+1)-1} - 3}{2}$ <p>But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is also true for $n = k + 1$. Since it is true for $n = 1$, it is true for all positive integers.</p>	<p>B1</p> <p>E1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>E1</p> <p>[6]</p>	<p>Showing use of $a_n = \frac{5 \times 3^{n-1} - 3}{2}$</p> <p>Assuming true for $n = k$</p> <p>a_{k+1}, using a_k and attempting to simplify</p> <p>Correct simplification to left hand expression.</p> <p>May be identified with a 'target' expression using $n = k + 1$</p> <p>Dependent on A1 and previous E1</p> <p>Dependent on B1 and previous E1</p>