Matrix Operations, Determinants and Inverses

Q1, (Jan 2006, Q1)

(i)	$2\mathbf{B} = \begin{pmatrix} 4 & -6 \\ 2 & 8 \end{pmatrix}$, $\mathbf{A} + \mathbf{C}$ is impossible,	B1 B1	
	$\mathbf{CA} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \\ 1 & 2 \end{pmatrix}$, $\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 6 \\ 0 & -2 \end{pmatrix}$	M1, A1 B1 **[5]**	**CA** 3×2 matrix M1
(ii)	$\mathbf{AB} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 4 & 5 \end{pmatrix}$ $\mathbf{BA} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 8 & 11 \end{pmatrix}$	M1	Or **AC** impossible, or student's own correct example. Allow M1 even if slip in multiplication
	$\mathbf{AB} \neq \mathbf{BA}$	E1 **[2]**	Meaning of commutative

Q2, (Jun 2007, Q1i)

(i)	$\mathbf{M}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1 **[2]**	Attempt to find determinant

Q3, (Jan 2007, Q9)

(i)	$\mathbf{M}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$ $\mathbf{N}^{-1} = \frac{1}{7}\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$	M1 A1 A1 [3]	Dividing by determinant One for each inverse c.a.o.
(ii)	$\mathbf{MN} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 1 & 4 \end{pmatrix}$	M1 A1	Must multiply in correct order
	$(\mathbf{MN})^{-1} = \frac{1}{21}\begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$	A1	Ft from **MN**
	$\mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{7}\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix} \times \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$	M1 A1	Multiplication in correct order Ft from (i)
	$= \frac{1}{21}\begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$ $= (\mathbf{MN})^{-1}$	A1 [6]	Statement of equivalence to $(\mathbf{MN})^{-1}$
iii)	$\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PI} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{P} = \mathbf{Q}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{PP}^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1}\mathbf{I} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow (\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$	M1 M1 M1 A1 [4]	$\mathbf{QQ}^{-1} = \mathbf{I}$ Correctly eliminate **I** from LHS Post-multiply both sides by $\mathbf{P}^{-1}$ at an appropriate point Correct and complete argument

Q4, (Jun 2009, Q1)

(i)	$\mathbf{M}^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$	M1 A1 [2]	Dividing by determinant
(ii)	$\frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 49 \\ 100 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 198 \\ 253 \end{pmatrix}$ $\Rightarrow x = 18, y = 23$	M1 A1(ft) A1(ft) [3]	Pre-multiplying by their inverse

Q5, (Jun 2015, Q1)

$\mathbf{M}^{-1} = \frac{1}{108}\begin{pmatrix} 21 & 3 \\ -8 & 4 \end{pmatrix}$	M1* M1* A1	Attempt to find $\mathbf{M}^{-1}$ or $108\mathbf{M}^{-1}$ Divide by their determinant, Δ, at some stage Correct determinant, (A0 for det **M**= $\frac{1}{108}$ stated, all other marks are available)
$\frac{1}{108}\begin{pmatrix} 21 & 3 \\ -8 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{5}{18} \\ \frac{1}{27} \end{pmatrix}$	M1 A1	Attempt to **pre**-multiply by inverse or by Δ $\mathbf{M}^{-1}$ Correct matrix multiplication (allow one slip)
$x = \frac{5}{18},\ y = \frac{1}{27}$, oe	A1dep*	For both, cao x and y must be specified, may be in column vectors **SC** answers only B1
	[6]	

Q6, (Jun 2010, Q2)

(i)	$2x - 5y = 9$ $3x + 7y = -1$	B1 B1 **[2]**	
(ii)	$\mathbf{M}^{-1} = \frac{1}{29}\begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix}$	M1 A1 **[2]**	Divide by determinant c.a.o.
	$\frac{1}{29}\begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 9 \\ -1 \end{pmatrix} = \frac{1}{29}\begin{pmatrix} 58 \\ -29 \end{pmatrix}$ $\Rightarrow x = 2,\ y = -1$	M1 A1(ft) **[2]**	Pre-multiply by their inverse For both

Q7, (Jun 2011, Q9)

(i)	$\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 3 & k \end{pmatrix}$	B2 **[2]**	- 1 each error
(ii)	$\mathbf{M}^{-1}$ does not exist for $2k+3=0$	M1	May be implied
	$k = \frac{-3}{2}$	A1	
	$\mathbf{M}^{-1} = \frac{1}{2k+3}\begin{pmatrix} k & 1 \\ -3 & 2 \end{pmatrix}$	B1	Correct inverse
	$\frac{1}{13}\begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 21 \end{pmatrix}$	M1	Attempt to pre-multiply by their inverse
	$= \begin{pmatrix} 2 \\ 3 \end{pmatrix}$	A1ft A1	Correct matrix multiplication c.a.o.
	$\Rightarrow x = 2,\ y = 3$	A1ft	At least one correct
		[7]	
iii)	There are no unique solutions	B1	
		[1]	
iv)	(A) Lines intersect (B) Lines parallel (C) Lines coincident	B1 B1 B1 **[3]**	