3x3 Matrix Determinants and Inverses (From OCR 4755)

Q1, (Jun 2008, Q5)

You are given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$.

(i) Calculate $\mathbf{AB}$. [3]

(ii) Write down $\mathbf{A}^{-1}$. [2]

Q2, (Jan 2009, Q10)

You are given that $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 1 & -1 & k \\ -2 & 7 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 5+k \\ -5 & 29 & 7 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} 42 & \alpha & 4k-8 \\ 10-5k & -16+29k & -12+6k \\ 0 & 0 & \beta \end{pmatrix}$.

(i) Show that $\alpha = 0$ and $\beta = 28 + 7k$. **[3]**

(ii) Find $\mathbf{AB}$ when $k = 2$. **[2]**

(iii) For the case when $k = 2$ write down the matrix $\mathbf{A}^{-1}$. **[3]**

(iv) Use the result from part **(iii)** to solve the following simultaneous equations. **[4]**

$$\begin{aligned} 3x + 4y - z &= 1 \\ x - y + 2z &= -9 \\ -2x + 7y - 3z &= 26 \end{aligned}$$

Q3, (Jan 2010, Q4)

You are given that if $\mathbf{M} = \begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ then $\mathbf{M}^{-1} = \frac{1}{k}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$.

Find the value of k. Hence solve the following simultaneous equations. **[6]**

$$\begin{aligned} 4x + z &= 9 \\ -6x + y + z &= 32 \\ 5x + 2y + 5z &= 81 \end{aligned}$$

Q4, (Jun 2013, Q3)

You are given that $\mathbf{N} = \begin{pmatrix} -9 & -2 & -4 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$ and $\mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & p & -6 \end{pmatrix}$.

(i) Find the value of p. **[2]**

(ii) Solve the equation $\mathbf{N}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$. **[4]**

Q5, (Jan 2011, Q9)

You are given that $\mathbf{A} = \begin{pmatrix} -2 & 1 & -5 \\ 3 & a & 1 \\ 1 & -1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2a+1 & 3 & 1+5a \\ -5 & 1 & -13 \\ -3-a & -1 & -2a-3 \end{pmatrix}$.

(i) Show that $\mathbf{AB} = (8+a)\mathbf{I}$. **[3]**

(ii) State the value of a for which $\mathbf{A}^{-1}$ does not exist. Write down $\mathbf{A}^{-1}$ in terms of a, when $\mathbf{A}^{-1}$ exists. **[3]**

(iii) Use $\mathbf{A}^{-1}$ to solve the following simultaneous equations. **[5]**

$$\begin{aligned} -2x + y - 5z &= -55 \\ 3x + 4y + z &= -9 \\ x - y + 2z &= 26 \end{aligned}$$

(iv) What can you say about the solutions of the following simultaneous equations? **[1]**

$$\begin{aligned} -2x + y - 5z &= p \\ 3x - 8y + z &= q \\ x - y + 2z &= r \end{aligned}$$

Q6, (Jun 2014, Q9)

You are given that $\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ -1 & \alpha & -1 \\ -2 & -1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3\alpha-1 & -8 & \alpha-3 \\ 5 & 1 & 2 \\ 2\alpha+1 & -5 & \alpha+3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} \gamma & 0 & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}$.

(i) Show that $\beta = 0$. **[2]**

(ii) Find γ in terms of α. **[2]**

(iii) Write down $\mathbf{A}^{-1}$ for the case when $\alpha = 2$. State the value of α for which $\mathbf{A}^{-1}$ does not exist. **[3]**

(iv) Use your answer to part **(iii)** to solve the following simultaneous equations.

$$\begin{aligned} x + 3y - z &= 25 \\ -x + 2y - z &= 11 \\ -2x - y + 3z &= -23 \end{aligned}$$

[5]

Q7, (Jun 2016, Q3)

You are given that $\mathbf{A} = \begin{pmatrix} \lambda & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -19 & 34 & -14 \\ 5 & -5 & 5 \\ -13 & 18 & -3 \end{pmatrix}$ and $\mathbf{AB} = \mu\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix.

(i) Find the values of λ and μ. **[4]**

(ii) Hence find $\mathbf{B}^{-1}$. **[2]**
