2x2 Matrix Transformations (From OCR 4755)

Q1, (Jan 2005, Q1)

You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -2 & 1 \end{pmatrix}$.

Find the inverse of **M**.

The transformation associated with **M** is applied to a figure of area 2 square units. What is the area of the transformed figure? [3]

Q2, (Jun 2006, Q1)

(i) State the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [1]

(ii) Write down the 2×2 matrix for rotation through 90° anticlockwise about the origin. [1]

(iii) Find the 2×2 matrix for rotation through 90° anticlockwise about the origin, followed by reflection in the x-axis. [2]

Q3, (Jan 2007, Q3)

The points A, B and C in the triangle in Fig. 3 are mapped to the points A′, B′ and C′ respectively under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.

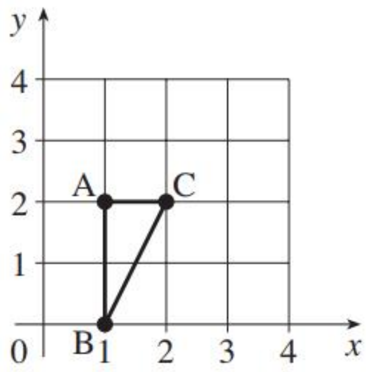

Fig. 3

(i) Draw a diagram showing the image of the triangle after the transformation, labelling the image of each point clearly. [4]

(ii) Describe fully the transformation represented by the matrix **M**. [3]

Q4, (Jun 2011, Q1)

(i) Write down the matrix for a rotation of 90° anticlockwise about the origin. **[1]**

(ii) Write down the matrix for a reflection in the line $y = x$. **[1]**

(iii) Find the matrix for the composite transformation of rotation of 90° anticlockwise about the origin, followed by a reflection in the line $y = x$. **[2]**

(iv) What single transformation is equivalent to this composite transformation? **[1]**

Q5, (Jan 2009, Q3)

Fig. 3 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

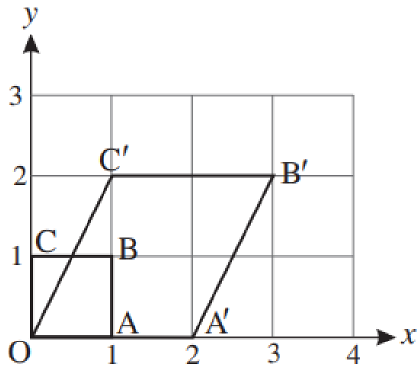

Fig. 3

(i) Write down the matrix **P** representing this transformation. **[1]**

(ii) The parallelogram OA′B′C′ is transformed by the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$. Find the coordinates of the vertices of its image, OA″B″C″, following this transformation. **[2]**

(iii) Describe fully the transformation represented by **QP**. **[2]**

Q6, (Jun 2010, Q9)

The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ represent transformations P and Q respectively.

(i) Describe fully the transformations P and Q. **[4]**

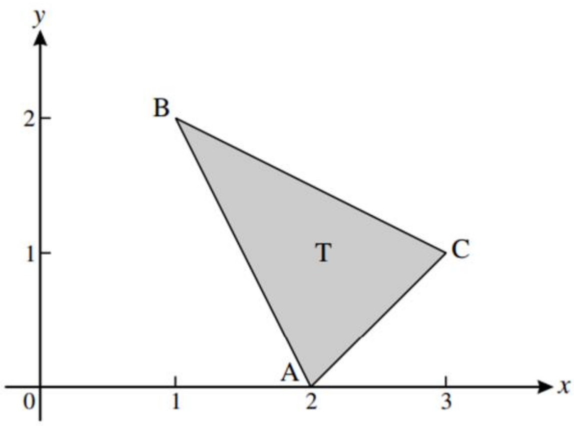

Fig. 9

Fig. 9 shows triangle T with vertices A (2, 0), B (1, 2) and C (3, 1).

Triangle T is transformed first by transformation P, then by transformation Q.

(ii) Find the single matrix that represents this composite transformation. **[2]**

(iii) This composite transformation maps triangle T onto triangle T′, with vertices A′, B′ and C′. Calculate the coordinates of A′, B′ and C′. **[2]**

T′ is reflected in the line $y = -x$ to give a new triangle, T″.

(iv) Find the matrix **R** that represents reflection in the line $y = -x$. **[2]**

(v) A single transformation maps T″ onto the original triangle, T. Find the matrix representing this transformation. **[4]**

Q7, (Jan 2011, Q2)

You are given that $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ -1 & 3 \end{pmatrix}$.

(i) The transformation associated with **M** is applied to a figure of area 3 square units. Find the area of the transformed figure. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and $\det \mathbf{M}^{-1}$. **[3]**

(iii) Explain the significance of $\det \mathbf{M} \times \det \mathbf{M}^{-1}$ in terms of transformations. **[2]**

Q8, (Jun 2014, Q2)

Fig. 2 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

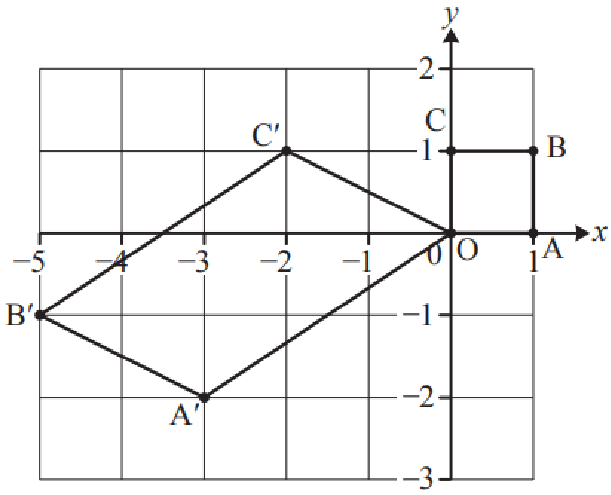

Fig. 2

(i) Write down the matrix **T** representing this transformation. **[2]**

The quadrilateral OA′B′C′ is reflected in the x-axis to give a new quadrilateral, OA″B″C″.

(ii) Write down the matrix representing reflection in the x-axis. **[1]**

(iii) Find the single matrix that will transform OABC onto OA″B″C″. **[2]**