

2x2 Matrix Transformations MS (From OCR 4755)

Q1, (Jan 2005, Q1)

Det $\mathbf{M} = 8$

$$\mathbf{M}^{-1} = \frac{1}{8} \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$$

Area = 16 square units

B1

B1

B1

[3]

Q2, (Jun 2006, Q2)

(i) Reflection in the x -axis.

B1

[1]

(ii) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

B1

[1]

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

M1

Multiplication of their matrices in the correct order

or B2 for correct matrix without working

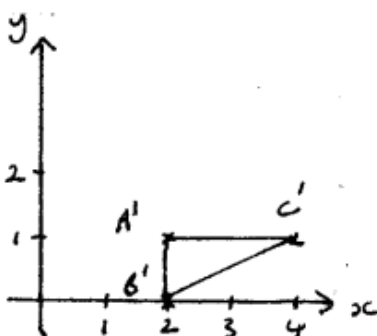
A1

c.a.o.

[2]

Q3, (Jan 2007, Q3)

(i)



B3

Points correctly plotted

B1

Points correctly labelled

$$\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \end{pmatrix}$$

ELSE

M1

Applying matrix to points

A1

Minus 1 each error

[4]

(ii) Stretch, factor 2 in x -direction, stretch factor half in y -direction.

B1

1 mark for stretch (withhold if rotation, reflection or translation mentioned incorrectly)

B1

1 mark for each factor and direction

B1

[3]

Q4, (Jun 2011, Q1)

(i)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1	Accept expressions in sin and cos
(ii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	
(iii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1 A1ft	
(iv)	Reflection in the x axis	B1	
			[5]

Q5, (Jan 2009, Q3)

(i)	$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$	B1	Applying matrix to column vectors, with a result.	
				[1]
(ii)	$\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$ $\Rightarrow A''=(4, 0), B''=(4, 6), C''=(0, 6)$	M1		
			[2]	
(iii)	Stretch factor 4 in x-direction. Stretch factor 6 in y-direction	B1 B1	Both factor and direction for each mark. SC1 for “enlargement”, not stretch.	
			[2]	

Q6, (Jun 2010, Q9)

<p>(i) P is a rotation through 90 degrees about the origin in a clockwise direction.</p> <p>Q is a stretch factor 2 parallel to the x-axis</p>	<p>B1 B1</p>	<p>Rotation about origin 90 degrees clockwise, or equivalent</p>
<p>ii)</p> $\mathbf{QP} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$	<p>B1 B1 [4]</p>	<p>Stretch factor 2 Parallel to the x-axis</p>
<p>iii)</p> $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 2 \\ -2 & -1 & -3 \end{pmatrix}$ <p>$A' = (0, -2), B' = (4, -1), C' = (2, -3)$</p>	<p>M1 A1 [2]</p>	<p>Correct order c.a.o.</p>
<p>iv)</p> $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	<p>M1</p>	<p>Pre-multiply by their QP - may be implied</p>
<p>v)</p> $\mathbf{RQP} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ $(\mathbf{RQP})^{-1} = \frac{-1}{2} \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	<p>A1(ft) [2]</p> <p>B1 B1 [2]</p> <p>M1 A1(ft)</p> <p>M1 A1 [4]</p>	<p>For all three points</p> <p>One for each correct column</p> <p>Multiplication of their matrices in correct order</p> <p>Attempt to calculate inverse of their RQP c.a.o.</p>

Q7, (Jan 2011, Q2)

<p>(i) $\det \mathbf{M} = 4 \times 3 - (-1) \times 0$</p> <p>Area = $12 \times 3 = 36$ square units</p>	<p>M1 A1 [2]</p>	<p>oe www</p>
<p>(ii) $\mathbf{M}^{-1} = \frac{1}{12} \begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix}$</p> <p>$\det \mathbf{M}^{-1} = \frac{1}{12}$</p>	<p>M1 A1</p> <p>B1 [3]</p>	<p>division by their $\det \mathbf{M}$ cao condone decimals 3sf or better</p> <p>cao condone decimal 3sf or better</p>
<p>iii) $\det \mathbf{M} \times \det \mathbf{M}^{-1} = 12 \times \frac{1}{12} = 1$</p> <p>The inverse 'undoes' the transformation, so the composite of M and its inverse must leave a shape unchanged, meaning the area scale factor of the composite transformation must be 1 and so the determinant is 1.</p>	<p>B1 E1 [2]</p>	<p>Seen or implied</p> <p>Any valid explanation involving transformations and unchanged area</p>

Q8, (Jun 2014, Q2)

(i)		$\begin{pmatrix} -3 & -2 \\ -2 & 1 \end{pmatrix}$	B1,B1 [2]
(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 [1]
(iii)		$\begin{pmatrix} -3 & -2 \\ 2 & -1 \end{pmatrix}$	B1,B1 [2]