The Argand Diagram and Loci Exam Questions (From OCR 4725)

Q1, (June 2006, Q6)

In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

Q2, (Jan 2007, Q4)

(i) Sketch, on an Argand diagram, the locus given by $|z - 1 + \mathrm{i}| = \sqrt{2}$. [3]

(ii) Shade on your diagram the region given by $1 \leqslant |z - 1 + \mathrm{i}| \leqslant \sqrt{2}$. [3]

Q3, (Jun 2007, Q8)

The loci C_1 and C_2 are given by $|z - 3| = 3$ and $\arg(z - 1) = \tfrac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \quad \text{and} \quad 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi.$$ [2]

Q4, (Jan 2009, Q10)

(i) Use an algebraic method to find the square roots of the complex number $2 + \mathrm{i}\sqrt{5}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + \mathrm{i}y$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \tfrac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

Q5, (Jan 2011, Q6)

(i) Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\mathrm{i}) = \tfrac{1}{4}\pi$. **[3]**

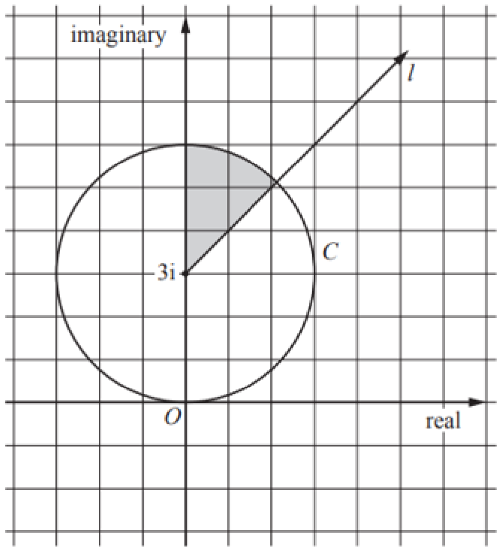
(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\mathrm{i}) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

Q6, (Jan 2012, Q6)

Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

Q7, (Jun 2013, Q6)



The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

Q8, (Jun 2015, Q5)

The loci C_1 and C_2 are given by $|z+2| = 2$ and $\arg(z+2) = \frac{5}{6}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[4]**

(ii) Find the complex number represented by the intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z+2| \leqslant 2 \text{ and } \tfrac{5}{6}\pi \leqslant \arg(z+2) \leqslant \pi.$$ **[2]**

Q8, (Jun 2016, Q6)

In an Argand diagram the points A and B represent the complex numbers $5+4\mathrm{i}$ and $1+2\mathrm{i}$ respectively.

(i) Given that A and B are the ends of a diameter of a circle C, find the equation of C in complex number form. **[4]**

The perpendicular bisector of AB is denoted by l.

(ii) Sketch C and l on a single Argand diagram. **[2]**

(iii) Find the complex numbers represented by the points of intersection of C and l. **[3]**