Motion in a Vertical Circle (From OCR 4730)

Q1, (Jun 2006, Q7)

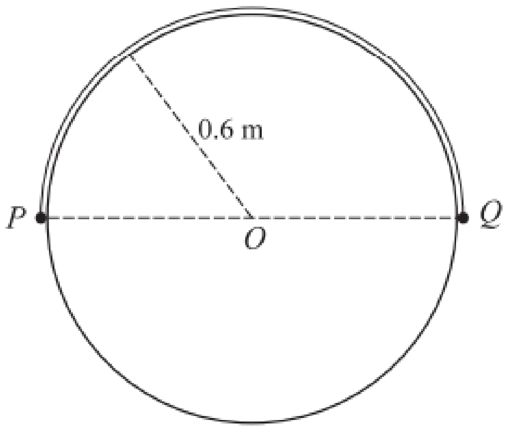


Fig. 1

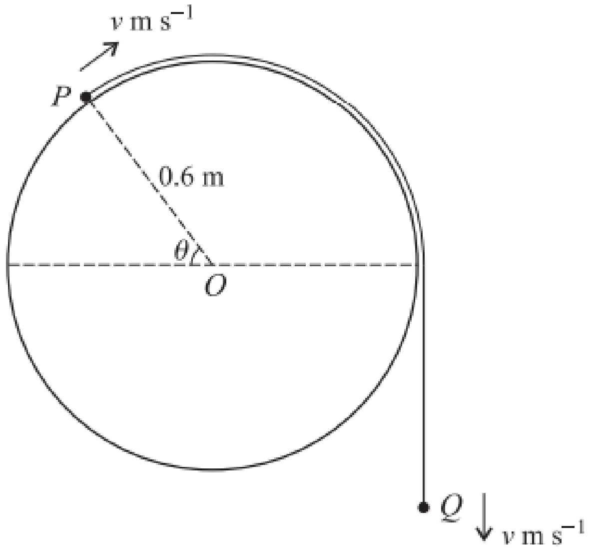


Fig. 2

A smooth horizontal cylinder of radius 0.6 m is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length 0.6π m has particles P and Q, of masses 0.3 kg and 0.4 kg respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, with the horizontal, the particles have speed $v\,\text{m s}^{-1}$ (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04\sin\theta.$$ **[5]**

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46\sin\theta - 3.36\theta) \text{ newtons.}$$

Hence find the value of θ for which the magnitude of the contact force is greatest. **[6]**

(iii) Find the transverse component of the acceleration of P in terms of θ. **[3]**

Q2, (Jun 2011, Q7)

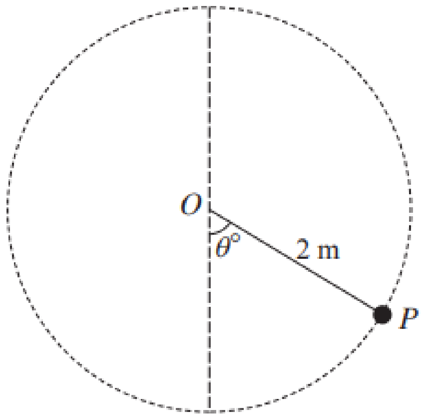
One end of a light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. P is projected horizontally from the point 0.8 m vertically below O with speed $5.6\,\text{m s}^{-1}$. P starts to move in a vertical circle with centre O. The speed of P is $v\,\text{m s}^{-1}$ when the string makes an angle θ with the downward vertical.

(i) While the string remains taut, show that $v^2 = 15.68(1 + \cos\theta)$, and find the tension in the string in terms of θ. **[7]**

(ii) For the instant when the string becomes slack, find the value of θ and the value of v. **[3]**

(iii) Find, in either order, the speed of P when it is at its greatest height after the string becomes slack, and the greatest height reached by P above its point of projection. **[4]**

Q3, (Jan 2008, Q6)

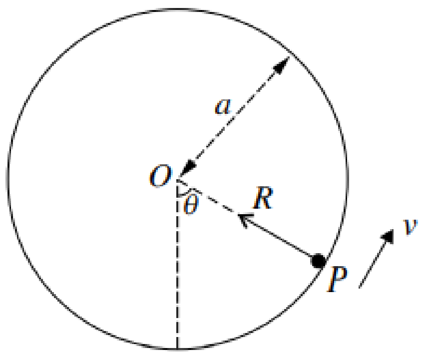


A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point O. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of P is $7\,\text{m s}^{-1}$.

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ. [8]

Q4, (Jun 2009, Q7)



A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \dfrac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

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Q5, (Jan 2010, Q6)

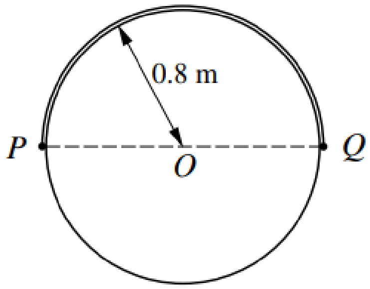


Fig. 1

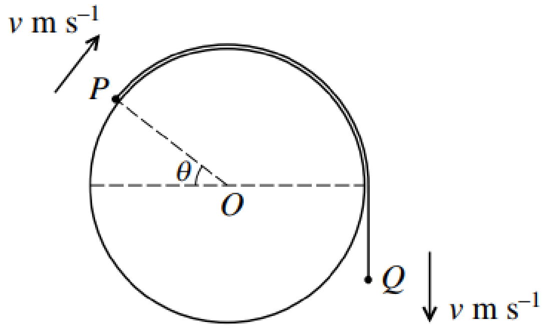


Fig. 2

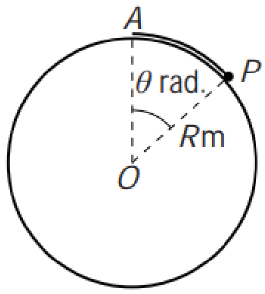
A light inextensible string of length 0.8π m has particles P and Q, of masses 0.4 kg and 0.58 kg respectively, attached to its ends. The string passes over a smooth horizontal cylinder of radius 0.8 m, which is fixed with its axis horizontal and passing through a fixed point O. The string is held at rest in a vertical plane perpendicular to the axis of the cylinder, with P and Q at opposite ends of the horizontal diameter of the cylinder through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v\,\text{m s}^{-1}$ (see Fig. 2).

(i) By considering the total energy of the system, obtain an expression for v^2 in terms of θ. **[5]**

(ii) Show that the magnitude of the force exerted on P by the cylinder is $(7.12\sin\theta - 4.64\theta)$ N. **[4]**

(iii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $1.53 < \alpha < 1.54$. **[4]**

Q6, (Jan 2012, Q7)



One end of a light elastic string, of natural length $\frac{2}{3}R$ m and with modulus of elasticity $1.2mg$ N, is attached to the highest point A of a smooth fixed sphere with centre O and radius R m. A particle P of mass m kg is attached to the other end of the string and is in contact with the surface of the sphere, where the angle AOP is equal to θ radians (see diagram).

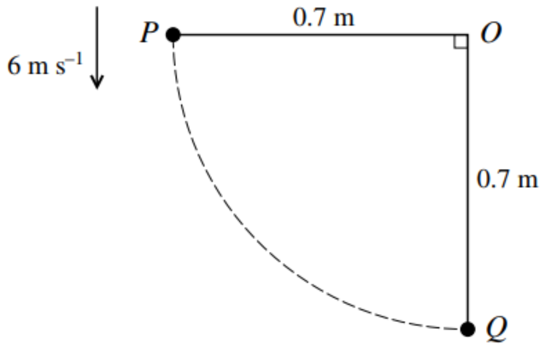
(i) Given that P is in equilibrium at the point where $\theta = \alpha$, show that $1.8\alpha - \sin\alpha - 1.2 = 0$. Hence show that $\alpha = 1.18$ correct to 3 significant figures. **[7]**

P is now released from rest at the point of the surface of the sphere where $\theta = \frac{2}{3}$, and starts to move downwards on the surface. For an instant when $\theta = \alpha$,

(ii) state the direction of the acceleration of P, **[1]**

(iii) find the magnitude of the acceleration of P. **[7]**

Q7, (Jun 2010, Q7)



A particle P is attached to a fixed point O by a light inextensible string of length 0.7 m. A particle Q is in equilibrium suspended from O by an identical string. With the string OP taut and horizontal, P is projected vertically downwards with speed 6 m s^{-1} so that it strikes Q directly (see diagram). P is brought to rest by the collision and Q starts to move with speed 4.9 m s^{-1}.

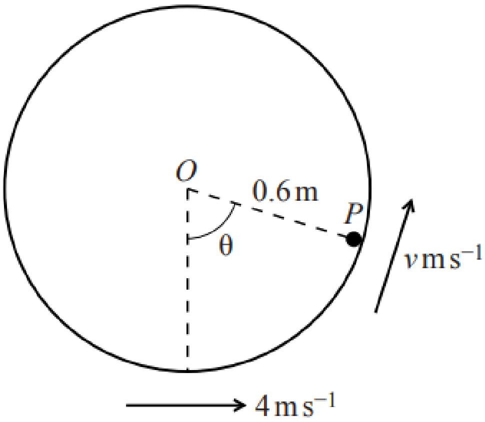
(i) Find the speed of P immediately before the collision. Hence find the coefficient of restitution between P and Q. **[3]**

(ii) Given that the speed of Q is v m s^{-1} when OQ makes an angle θ with the downward vertical, find an expression for v^2 in terms of θ, and show that the tension in the string OQ is $14.7m(1 + 2\cos\theta)$ N, where m kg is the mass of Q. **[6]**

(iii) Find the radial and transverse components of the acceleration of Q at the instant that the string OQ becomes slack. **[4]**

(iv) Show that $V^2 = 0.8575$, where V m s^{-1} is the speed of Q when it reaches its greatest height (after the string OQ becomes slack). Hence find the greatest height reached by Q above its initial position. **[4]**

Q8, (Jun 2012, Q4)

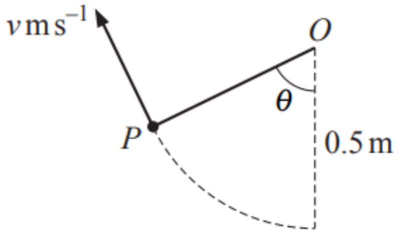


A hollow cylinder is fixed with its axis horizontal. The inner surface of the cylinder is smooth and has radius 0.6 m. A particle P of mass 0.45 kg is projected horizontally with speed 4 m s^{-1} from the lowest point of a vertical cross-section of the cylinder and moves in the plane of the cross-section, which is perpendicular to the axis of the cylinder. While P remains in contact with the surface, its speed is v m s^{-1} when OP makes an angle θ with the downward vertical at O, where O is the centre of the cross-section (see diagram). The force exerted on P by the surface is R N.

(i) Show that $v^2 = 4.24 + 11.76\cos\theta$ and find an expression for R in terms of θ. **[6]**

(ii) Find the speed of P at the instant when it leaves the surface. **[4]**

Q9, (Jun 2015, Q7)



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is projected horizontally from the point 0.5 m below O with speed $u\,\mathrm{m\,s^{-1}}$. When the string makes an angle of θ with the downward vertical the particle has speed $v\,\mathrm{m\,s^{-1}}$ (see diagram).

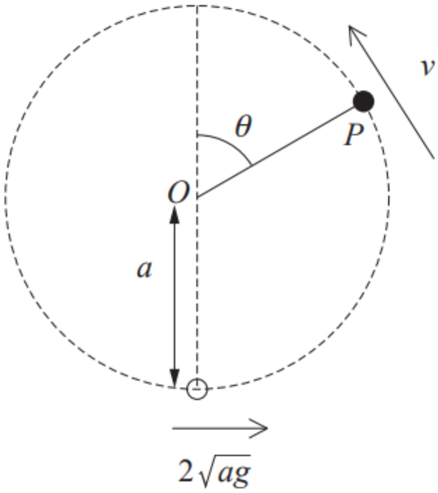
(i) Show that, while the string is taut, the tension, T N, in the string is given by

$$T = 5.88\cos\theta + 0.4u^2 - 3.92.$$ **[5]**

(ii) Find the least value of u for which the particle will move in a complete circle. **[3]**

(iii) If in fact $u = 3.5\,\mathrm{m\,s^{-1}}$, find the speed of the particle at the point where the string first becomes slack. **[4]**

Q10, (Jun 2016, Q5)



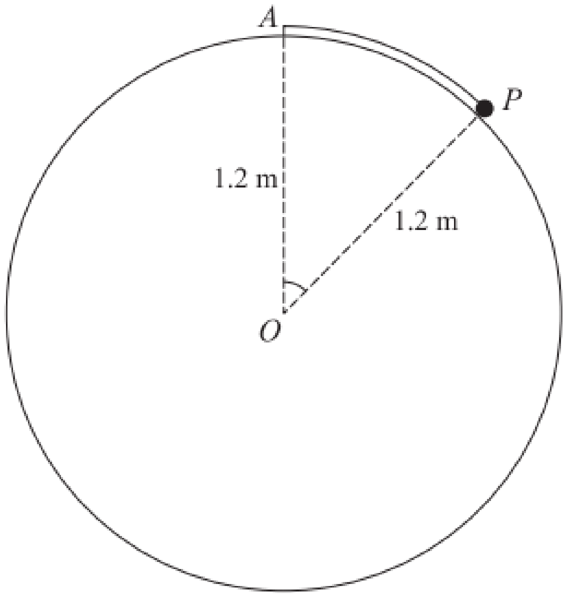
One end of a light inextensible string of length a is attached to a fixed point O. A particle P of mass m is attached to the other end of the string and hangs at rest. P is then projected horizontally from this position with speed $2\sqrt{ag}$. When the string makes an angle θ with the upward vertical P has speed v (see diagram). The tension in the string is T.

(i) Find an expression for T in terms of m, g and θ, and hence find the height of P above its initial level when the string becomes slack. **[6]**

P is now projected horizontally from the same initial position with speed U.

(ii) Find the set of values of U for which the string does not remain taut in the subsequent motion. **[5]**

Q11, (Jan 2007, Q7)



The diagram shows a particle P of mass 0.5 kg attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 1.2 m. The string has natural length 0.6 m and modulus of elasticity 6.86 N. P is released from rest at a point on the surface of the sphere where the acute angle AOP is at least 0.5 radians.

(i) **(a)** For the case angle $AOP = \alpha$, P remains at rest. Show that $\sin\alpha = 2.8\alpha - 1.4$. [4]

(b) Use the iterative formula

$$\alpha_{n+1} = \frac{\sin\alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find α correct to 2 significant figures. [2]

(ii) Given instead that angle $AOP = 0.5$ radians when P is released, find the speed of P when angle $AOP = 0.8$ radians, given that P is at all times in contact with the surface of the sphere. State whether the speed of P is increasing or decreasing when angle $AOP = 0.8$ radians. [7]