

Coefficient of Restitution MS (From OCR 4729)

Q1, (Jan 2007, Q2)

$e = 1 = (y-x)/4$	B1		or $\frac{1}{2} \times 0.2x^2 + \frac{1}{2} \times 0.1y^2 =$	
$0.8 = 0.2x + 0.1y$	B1		$\frac{1}{2} \times 0.2x^2$ (B1/B1 for any 2)	
solving sim. equ.	M1		not if poor quad. soln.	
$x = 4/3$ only	A1	4		4

Q2, (Jan 2007, Q4)

(i)	$\frac{1}{2} \times 80 \times 5^2$ or $\frac{1}{2} \times 80 \times 2^2$	either KE	B1		1000/160	
	70×25		B1		1750	
	$80 \times 9.8 \times 25 \sin 20^\circ$		B1		6703.6	
	$WD = \frac{1}{2} \times 80 \times 5^2 - \frac{1}{2} \times 80 \times 2^2 + 70 \times 25 + 80 \times 9.8 \times 25 \sin 20^\circ$		M1		4 parts	
	9290		A1	5		
(ii)	$P \cos 30^\circ \times 25$		B1		or $a = 0.42$	
	$P \cos 30^\circ \cdot 25 = 9290 / P \cos 30^\circ - 70 - 80 \times 9.8 \sin 20^\circ = 80a$		M1			
	$P = 429$ /if P found 1 st then $P \cos 30^\circ \times 25 = 9290$ ok		A1	3		8

Q3, (Jun 2005, Q4)

(i)	$5m = mu + 4m$	M1		cons. of mom.	
	$u = 1$	A1			
	$e = (2-1)/5$	M1			
	$e = 0.2$	A1	4		
(ii)	$I = 4m$	B1			
	\rightarrow	B1	2	to the right	
(iii)	$4m = 5mv$	M1			
	$v = 0.8$	A1			
	< 1	B1	3		9

Q4, (Jun 2006, Q8)

(i)	$10 = 4 + m \cdot x$ $e = \dots$ or rationale for $x = 2$ $m = 3$	M1 M1 A1	3	conservation of momentum	
(ii)	$v = 6$ $e = 4/5$ or 0.8	B1 M1 A1	3	allow sign errors for M mark watch out for lost minuses	
(iii)	$10 - 5 = 2x + y$ ($5 = -2a + b$) $(-5 = 2c + d)$ $e = 0.8 = (y-x)/10$ $y = x + 8$ ($a + b = 8$) ($c - d = 8$) $x = -1$ ($a=1$) ($c=1$) $y = 7$ ($b=7$) ($d=-7$) $\frac{1}{2} \cdot 2.5^2 + \frac{1}{2} \cdot 1.5^2 - \frac{1}{2} \cdot 2.1^2 - \frac{1}{2} \cdot 1.7^2$ 12 J	M1 A1 M1 A1 A1 A1 M1 A1	8	look for consistency or 1 in opp. direction to 1st K.E. lost. Must be 4 parts (37.5 - 25.5)	14

Q5, (Jun 2009, Q6)

(i)	$I = 0.9 = 6 \times 0.2 - v \times 0.2$ $v = 1.5$	M1 A1 A1 3	needs to be mass 0.2
(ii)	$0.6 = (c - b) / 6$ $6 \times 0.2 = 0.2b + 0.1c$ $b = 2.8$ $0.4 \times 5 + 0.2 \times 1.5 = 0.4a + 0.2 \times 6 \quad \text{or}$ $I = 0.9 = -0.4a - - 0.4 \times 5$ $a = 2.75$ $2.75 < 2.8$ no further collision	M1 A1 M1 A1 A1 M1 A1 A1 M1 A1 10	restitution (allow 1.5 for M1) momentum (allow 1.5 for M1) 1st collision (needs their 1.5 for M1) compare v 's of A and B (calculated)

Q6, (Jan 2011, Q7)

(i)		$b + a = 1.8e$	M1	Uses restitution
		$0.7b - 0.2a = 0.2 \times 1.8$	A1	$b - a = 1.8e$
		$b = 0.4(1+e)$	M1	Uses momentum
		$a = 1.4e - 0.4$	A1	$0.7b + 0.2a = 0.2 \times 1.8$, signs consistent with first eqn
		$1.4e - 0.4 > 0.4 + 0.4e$	M1	Solves 2 simultaneous equations (eliminate a or b)
		$e > 0.8$	A1	$a = 0.4 - 1.4e$
OR	Last 5 marks	Using $a > b$	A1	Using $a > b$, correct signs in a essential
		$a > 0.72$	M1	correct signs in a essential
		$b > 0.72$	A1	
		$1.8e > 0.72 + 0.72$	M1	
		$e > 0.8$	A1	
OR	Last 5 marks	Using $a = b$ to find a or b	M1	
		a (or b) = $0.9e$ and a (or b) = 0.72	A1	
		$e = 0.8$	A1	
		Convincing argument for correct inequality	M1	
		$e > 0.8$	A1	
OR	Last 5 marks	$a = 1.4e - 0.4$ or $b = 0.4(1+e)$	M1	Solves 2 simultaneous equations (eliminate a or b)
		Using $a > b$	A1	aef or multiples thereof
		$a > 0.9e$ or $b < 0.9e$	M1	correct signs in a essential
		$e > 0.8$	A1	aef or multiples thereof
			A1	

(ii)	$c - (\pm 0.25) = 1 \times 0.75$ $c = 0.5, 1$ $0.75 \times 0.7 = 0.25 \times 0.7 + m(x1)$ OR $0.75 \times 0.7 = -0.25 \times 0.7 + 0.5m$ $m = 0.35$ (from first equation) $m = 1.4$ (from second equation)	M1 A1A1 M1 A1 A1 [6]	Uses restitution with $e = 1$, either Or 0.75 ± 0.25 Uses momentum conservation with correct combination of sign and c value OR $mx(0.75 \pm 0.25) \pm 0.7 \times 0.25 = 0.75 \times 0.7$
	OR $\frac{1}{2} \times 0.7 \times 0.75^2 = \frac{1}{2} \times 0.7 \times 0.25^2 + \frac{1}{2} mc^2$ $0.7 \times 0.75 = 0.7 \times (+/- 0.25) + mc$ Solving simultaneous equations $m = 0.35$ $m = 1.4$	B1 M1 A1 M1 A1 A1	$\frac{1}{2}$ may not be seen At least one momentum equation $mc = 0.35$ and 0.7

Q7, (Jan 2013, Q3)

(i)	$a = g \sin 30$ $1 + u = 0.4(2 + 2g \sin 30)$ $u = 3.72 \text{ ms}^{-1}$	B1 M1 A1 A1 [4]	Using NEL with u_A from $cv(a)$, $u_A \neq 0$ cwo
(ii)	Use $v^2 = u^2 - 2(g \sin 30)s$ $s = 1.41 \text{ m}$	M1 A1 [2]	Using $v = 0$, $cv(a)$ from (i) or correct a SC If a not found in (i), allow $a = g$ for M1A0.
(iii)	Use of conservation of momentum $0.5 \times 2g \sin 30 - 2m = m - 0.5 \times 3.72$ $m = 2.25$	M1 A1ft A1 [3]	Using $cv(a)$ ft $cv(u)$ from (i) Aef(raction) eg $2^{19/75}$ or $169/75$

Q8, (Jun 2014, Q6)

(i)	$(2m)(4) - (3m)(2) = 2mv_A + 3mv_B$ $(v_B - v_A)/(4 - -2) = 0.4$ <p>Speed $A = 1.04 \text{ m s}^{-1}$, Speed $B = 1.36 \text{ m s}^{-1}$</p>	<p>*M1 A1 *M1 A1 Dep**M1 A1 [6]</p>	<p>Attempt at use of conservation of momentum</p> <p>Attempt at use of coefficient of restitution</p> <p>Solving for v_A and v_B</p> <p>Final answers must be positive</p>
(ii)	<p>Energy before = $\frac{1}{2}(2m)(4^2) + \frac{1}{2}(3m)(2^2)$ Energy after = $\frac{1}{2}(2m)(1.04^2) + \frac{1}{2}(3m)(1.36^2)$</p> <p>$22m - 3.856m$</p> <p>$18.1m$</p>	<p>B1ft B1ft M1 A1 [4]</p>	<p>Energy before or Loss in A's KE Energy after or Loss in B's KE</p> <p>Difference of total OR sum of differences (total kinetic energy must decrease)</p> <p>$18.144m$ (Exact)</p>
OR	$\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (1 - e^2) A^2$ $\frac{1}{2} \frac{(2m)(3m)}{2m + 3m} (1 - 0.4^2)(4 + 2)^2$ <p>$18.1m$</p>	<p>*B1 Dep*M1 A1 A1 [4]</p>	<p>Loss of kinetic energy formula, where A = approach speed</p> <p>Substitution of values into quoted formula</p> <p>$18.144m$ (Exact)</p>
(iii)	<p>$2m(4) - 2m(-1.04) = 2.52$ $m = 0.25$</p>	<p>M1 A1ft A1 [3]</p>	<p>Attempt at change in momentum and equate to impulse. Must use $2m$ or $3m$ Or $3m(2) - 3m(-1.36) = 2.52$ Exact</p>

Q9, (Jun 2013, Q6)

<p>(i)</p>	$4 - 4(1 - e + e^2) = -e(u - 4)$ $u = 4e$ $mu + 0.2 \times 4 = 0.2 \times 4(1 - e + e^2) + 4m$ $m = 0.2e$	<p>M1 A1 A1 M1 A1 A1 [6]</p>	<p>Use of restitution, may have sign errors, must be correct ratio (v/u) oe Use of conservation of momentum oe</p>
<p>(ii)</p>	<p>Valid method to find e that gives the least speed Get $e = \frac{1}{2}$</p> $\frac{1}{2} \times 0.2 \times 4^2 + \frac{1}{2} \times 0.1 \times 2^2 - (\frac{1}{2} \times 0.2 \times 3^2 + \frac{1}{2} \times 0.1 \times 4^2)$ <p>(+/-) 0.1 J</p>	<p>M1 A1 M1 A1 A1 [5]</p>	<p>Differentiate v_A and equate to 0 or complete the square on v_A www Difference of KE with 4 terms Must have found the value of e from a legitimate method. www SCM1A1 Loss of KE = $8e(1 - e)^3/5$ or $8e(1 - 3e + 3e^2 - e^3)/5$ or $8e/5 - 24e^2/5 + 24e^3/5 - 8e^4/5$</p>
<p>(iii)</p>	$0.2e(4 - 4e) = 0.192 \text{ or } 0.2(4 - (4 - 4e + 4e^2)) = 0.192$ <p>Solve three term QE in e $e = 0.4 \text{ or } 0.6$</p>	<p>*M1 A1 dep*M1 A1 [4]</p>	<p>Attempt to use impulse = change in momentum on one particle method should lead to 2 real values for e For both</p>

Q10, (Jun 2015, Q5)

(i)	$v^2 = 5^2 + 2g(1.6)$ $0.7 \times 7.507... (= 5.255...)$ $(0.7 \times cv(v))^2 = 2gh$ $h = 1.41 \text{ m}$	B1 B1 M1 A1 [4]	Complete method to find $v (= 7.507...)$ $0.7 \times cv(v)$, but not $cv(v) = 5$; may be seen in (ii) Complete method to find h , with final speed 0; allow $cv(v) = 5$ for method Exact 1.409
(ii)	$0.2(7.507...)(0.7) - (-0.2)(7.507...)$ Impulse = 2.55 N s, upwards	M1 A1ft A1 [3]	Change in momentum found, with relevant velocities ie $cv(v)$ and $0.7 \times cv(v)$ but not $cv(v) = 5$ This may be negative; ft on their v found in (i) (2.5524...) Must have direction also.
(iii)	$0.2(9.8)(1.6) + \frac{1}{2}(0.2)(5^2) - 0.2(9.8)(cv(h))$ OR $\frac{1}{2}(0.2)(7.507...^2) - \frac{1}{2}(0.2)(0.7 \times 7.507...)^2$ Loss of energy = 2.87 J	M1 A1ft A1 [3]	Change in energy found, all energy terms needed and no extra terms This may be negative (2.87436 exact); art 2.87; allow -2.87

