

Normal Distribution Exam Questions

Q1, (Edexcel 6683, Jun 2005, Q6)

(a)	$M \sim N(155, 3.5^2)$ $P(M > 160) = P\left(z > \frac{160-155}{3.5}\right)$ $= P(z > 1.43)$ $= 0.0764$	standardising $\pm(160-155), \sigma, \sigma^2, \sqrt{\sigma}$	M1 A1 A1 (3)
(b)	$P(150 \leq M \leq 157) = P(-1.43 \leq z \leq 0.57)$ $= 0.7157 - (1 - 0.9236)$ $= 0.6393$ special case : answer only B0 B0 M1 A1	awrt -1.43, 0.57 p>0.5 0.6393 - 0.6400 4dp	B1 B1 M1 A1 (4)
(c)	$P(M \leq m) = 0.3 \Rightarrow \frac{m-155}{3.5} = -0.5244$ $m = 153.2$	-0.5244 att stand = z value for A1 may use awrt to - 0.52. cao	B1 M1 A1 A1 (4)

Q2, (Edexcel 6683, Jun 2007, Q6)

(a)	$P(X > 25) = P\left(Z > \frac{25-20}{4}\right)$ $= P(Z > 1.25)$ $= 1 - 0.8944$ $= 0.1056$	M1 M1 A1 (3)
(b)	$P(X < 20) = 0.5$ so $P(X < d) = 0.5 + 0.4641 = 0.9641$ $P(Z < z) = 0.9641, z = 1.80$ $\frac{d-20}{4} = 1.80$ $d = 27.2$	B1 B1 M1 A1 (4)

Total 7 marks

Q3, (Edexcel 6683, Jan 2008, Q6)

(a)	200 or 200g			B1	
(b)	$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.)			M1	(1)
				A1	
	$Z = (\pm) \frac{210 - 200}{\sigma}$			M1	
	$\frac{10}{\sigma} = 0.8416$		0.8416	B1	
	$\sigma = 11.882129\dots$		AWRT 11.9	A1	
(c)	$P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ $= P(Z < -1.6832)$ $= 1 - 0.9535$ $= 0.0465$ or AWRT 0.046			M1	(5)
				M1	
				A1	
					(3)
					Total 9 marks

Q4, (OCR 4733, Jan 2008, Q1)

$\frac{80 - \mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$	M1	Standardise once with Φ^{-1} , allow σ^2 , cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong z, <i>not</i> 1-1.645 Solve two standardised equations μ , a.r.t 58.7 σ , a.r.t. 12.9 [<i>not</i> σ^2] [σ^2 : M1B1A0M1A1A0]
$\frac{\mu - 50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$	B1	
Solve simultaneously	A1	
$\mu = 58.7, \sigma = 12.9$	A1	
	M1	
	A1	
	6	

Q5, (Edexcel 6683, Jun 2008, Q7)

(a)	$z = \frac{53 - 50}{2}$ $P(X > 53) = 1 - P(Z < 1.5)$ $= 1 - 0.9332$ $= 0.0668$	Attempt to standardise	M1	
		1-probability required can be implied	B1	
			A1	[3]
(b)	$P(X \leq x_0) = 0.01$ $\frac{x_0 - 50}{2} = -2.3263$ $x_0 = 45.3474$		M1	
		awrt 45.3 or 45.4	M1B1	
			M1A1	[5]
(c)	$P(2 \text{ weigh more than } 53\text{kg and } 1 \text{ less}) = 3 \times 0.0668^2 (1 - 0.0668)$ $= 0.012492487\dots$		B1M1A1ft	
		awrt 0.012	A1	[4]
				Total 12

Q6, (Edexcel 6683, Jan 2010, Q7)

(a)		bell shaped, must have inflexions 154,172 on axis 5% and 30%	B1 B1 B1 (3)
(b)	$P(X < 154) = 0.05$ $\frac{154 - \mu}{\sigma} = -1.6449$ or $\frac{\mu - 154}{\sigma} = 1.6449$ $\mu = 154 + 1.6449\sigma$ **given**		M1 B1 A1 cso (3)
(c)	$172 - \mu = 0.5244\sigma$ or $\frac{172 - \mu}{\sigma} = 0.5244$ Solving gives $\sigma = 8.2976075$ (awrt 8.30) and $\mu = 167.64873$ (awrt 168)	(allow $z = 0.52$ or better here but must be in an equation)	B1 M1 A1 A1 (4)
(d)	$P(\text{Taller than 160cm}) = P\left(Z > \frac{160 - \mu}{\sigma}\right)$ $= P(Z < 0.9217994)$ $= 0.8212$ awrt 0.82		M1 B1 A1 (3) Total [13]

Q7, (OCR 4733, Jun 2006, Q3)

(i)	$(140 - \mu)/\sigma = -2.326$ $(300 - \mu)/\sigma = 0.842$ Solve to obtain: $\mu = 257.49$ $\sigma = 50.51$	M1 B1 A1√ M1 A1 A1 6	One standardisation equated to Φ^{-1} , allow "1-", σ^2 Both 2.33 and 0.84 at least, ignore signs Both equations completely correct, √ on their z Solve two simultaneous equations to find one variable μ value, in range [257, 258] σ in range [50.4, 50.55]
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Q8, (Edexcel 6683, Jun 2012, Q6)

(a)	$[z =] \pm \left(\frac{150 - 162}{7.5} \right)$ $[z =] -1.6$ $[P(F > 150) = P(Z > -1.6) =] = 0.9452(0071\dots)$	M1 A1 A1	awrt 0.945	(3)
(b)	$(\pm) \frac{s - 162}{7.5} = 0.2533(47\dots)$ $s = 163.9$	B1 M1 A1	$z = \pm 0.2533$ (or better seen) awrt 164	(3)
(c)	$\frac{162 - \mu}{9} = -1.2815515\dots$ $\mu = 173.533\dots$	B1 M1 A1 A1	$z = \pm 1.2816$ (or better seen) awrt 174	(4)
				[10]

Q9, (Edexcel 6683, Jun 2013(R), Q4)

(a)	$\left[P(M < 145) = \right] P\left(Z < \frac{145 - 150}{10} \right)$ $= P(Z < -0.5) \text{ or } P(Z > 0.5)$	M1 A1 A1	= awrt 0.309	(3)
(b)	$\left[P(B > 115) = 0.15 \Rightarrow \right] \frac{115 - 100}{d} = 1.0364$	M1B1A1 A1	$d = \underline{\underline{14.5}}$ (Calc gives 1.036433...) (Calc gives 14.4727...)	(4)
(c)	$\left[P(X > \mu + 15 \mid X > \mu - 15) = \right] \frac{P(X > \mu + 15)}{P(X > \mu - 15)}$	M1 A1 A1	$= \frac{0.35}{1 - 0.35}$ $= \underline{\underline{\frac{7}{13}}} \text{ or awrt } \underline{\underline{0.538}}$	(3)
				[10]

Q10, (Edexcel 6683, Jun 2013, Q6)

(a)	<p>[Let X be the amount of beans in a tin. $P(X < 200) = 0.1$]</p> $\frac{200 - \mu}{7.8} = -1.2816$ <p style="text-align: right;">[calc gives 1.28155156...]</p> $\mu = 209.996\dots$ <p style="text-align: right;">awrt 210</p>	<p>M1 B1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
(b)	$P(X > 225) = P\left(Z > \frac{225 - "210"}{7.8}\right)$ $= P(Z > 1.92) \quad \text{or} \quad 1 - P(Z < 1.92) \quad \text{(allow 1.93)}$ $= 1 - 0.9726 = 0.0274 \quad \text{(or better)} \quad \text{[calc gives 0.0272037...]}$ $= 0.0274$ <p style="text-align: right;">= awrt <u>2.7%</u> allow <u>0.027</u></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
(c)	<p>[Let Y be the new amount of beans in a tin]</p> $\frac{210 - 205}{\sigma} = 2.3263 \quad \text{or} \quad \frac{200 - 205}{\sigma} = -2.3263 \quad \text{[calc gives 2.3263478...]}$ $\sigma = \frac{5}{2.3263}$ $\sigma = 2.15 \quad (2.14933\dots)$	<p>M1 B1</p> <p>dM1</p> <p>A1</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">(10 marks)</p>

Q11, (OCR 4733, Jan 2012, Q3)

$$\mu = 60$$

$$\frac{63.8 - \mu}{\sigma} = \Phi^{-1}(0.9) = 1.282$$

$$\sigma = 2.96(4)$$

$$1 - \Phi\left(\frac{65 - 60}{2.964}\right) = 1 - \Phi(1.687)$$

$$= \mathbf{0.0458}$$

B1	$\mu = 60$ stated or implied, can be written down
M1	Standardise 63.8 or 56.2 with σ , allow $\sqrt{\quad}$ or cc errors, equate to
B1	Φ^{-1}
	1.282 (or 1.281 or 1.28) seen
A1	σ , in range [2.96, 2.97], can be implied by what follows, <i>not</i> σ^2
M1	Standardise 65 with their μ and σ , allow $\sqrt{\quad}$ or cc errors
A1	Final answer, a.r.t. 0.046, c.w.o.
[6]	

Q12, (OCR 4733, Jun 2015, Q1)

$$\frac{150 - \mu}{\sigma} = 2.00$$

$$\frac{143 - \mu}{\sigma} = -1.5$$

Solve to get

$$\mu = 146, \sigma = 2$$

M1	Standardise with σ, μ at least once, ignore cc, $\sqrt{\quad}$ errors, equate to z	z not used, e.g. equated to 0.0228 and 0.9332 or 0.5092 and 0.8246: max M0M1
A1	Both LHS and signs of RHS correct	One z , one not: M1A0B0
B1	Both z -values correct to 3 SF	
M1	Correct method for solution	Withhold if elimination done wrongly
A1	$\mu \in [145.95, 146.05]$ www	$\sqrt{\sigma}$ or σ^2 : can get M1A0B1M1A1A0
A1	$\sigma \in [1.995, 2.005]$ or $\sigma^2 = 4$ www	cc: M1A0B1M1A0A0
6		

Q13, (OCR 4733, Jun 2016, Q2)

$$\frac{1.03 - 1.00}{\sigma} = 1.645$$

$$[\sigma = 0.0182... \approx \frac{6}{329}]$$

$$1 - \Phi\left(\frac{1.05 - 1.03}{\sigma}\right) = 1 - \Phi(1.0966)$$

$$= 1 - 0.8635 = \mathbf{0.1365} \text{ or } 13.6(5)\%$$

M1dep*	Standardise and equate to Φ^{-1} , allow wrong sign, σ^2 , 1-, cc etc
A1	All correct apart possibly from value of Φ^{-1}
B1	1.645 seen anywhere, allow -1.645, can be implied
*M1	Solve to find σ , or eliminate σ , dependent on first M1
M1	Standardise with $\mu = 1.03$, use Φ , answer < 0.5 , allow $\sqrt{\quad}$ errors
A1	6 Final answer in range [0.1355, 0.137] or [13.55%, 13.7%], must be from positive σ , not from σ^2
	0.1333 from $\sigma = 0.018$ is 5+A0