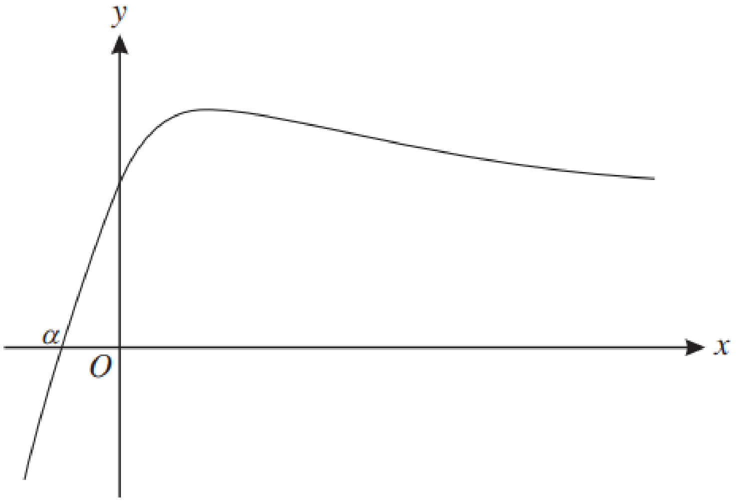
The Newton-Raphson Method (From OCR 4726)

Q1, (Jan 2006, Q2)

Use the Newton-Raphson method to find the root of the equation $e^{-x} = x$ which is close to $x = 0.5$. Give the root correct to 3 decimal places. [5]

Q2, (Jan 2008, Q5)



The diagram shows the curve with equation $y = xe^{-x} + 1$. The curve crosses the x-axis at $x = \alpha$.

(i) Use differentiation to show that the x-coordinate of the stationary point is 1. [2]

α is to be found using the Newton-Raphson method, with $f(x) = xe^{-x} + 1$.

(ii) Explain why this method will not converge to α if an initial approximation x_1 is chosen such that $x_1 > 1$. [2]

(iii) Use this method, with a first approximation $x_1 = 0$, to find the next three approximations x_2, x_3 and x_4. Find α, correct to 3 decimal places. [5]

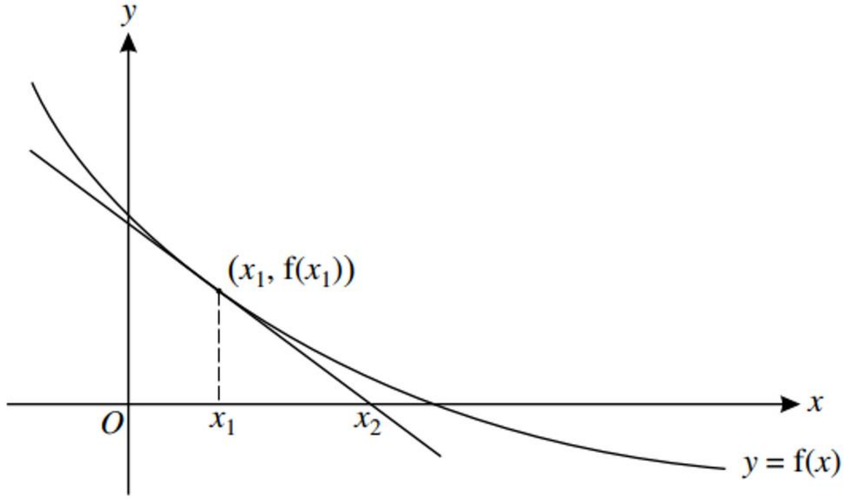
Q3, (Jun 2008, Q6i,ii)

It is given that $f(x) = 1 - \frac{7}{x^2}$.

(i) Use the Newton-Raphson method, with a first approximation $x_1 = 2.5$, to find the next approximations x_2 and x_3 to a root of $f(x) = 0$. Give the answers correct to 6 decimal places. [3]

(ii) The root of $f(x) = 0$ for which x_1, x_2 and x_3 are approximations is denoted by α. Write down the exact value of α. [1]

Q4, (Jan 2010, Q3i,ii)

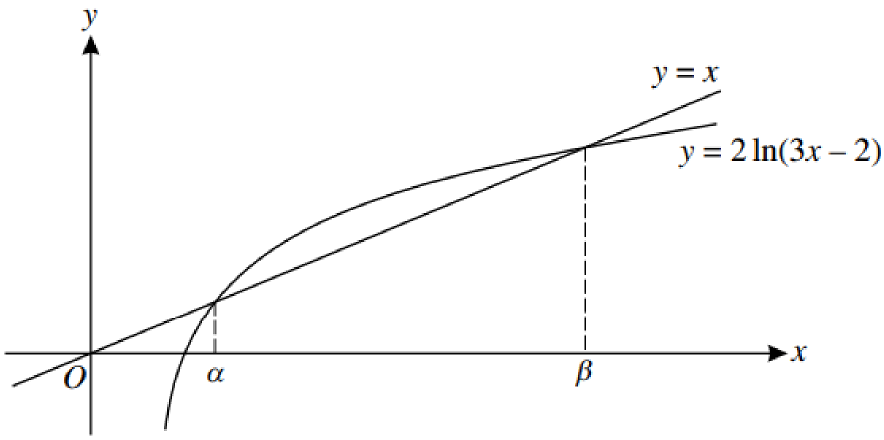


A curve with no stationary points has equation $y = f(x)$. The equation $f(x) = 0$ has one real root α, and the Newton-Raphson method is to be used to find α. The tangent to the curve at the point $(x_1, f(x_1))$ meets the x-axis where $x = x_2$ (see diagram).

(i) Show that $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. **[3]**

(ii) Describe briefly, with the help of a sketch, how the Newton-Raphson method, using an initial approximation $x = x_1$, gives a sequence of approximations approaching α. **[2]**

Q5, (Jun 2010, Q7iii,iv)



The line $y = x$ and the curve $y = 2\ln(3x - 2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

(iii) Show that the equation $x = 2\ln(3x - 2)$ can be rewritten as $x = \frac{1}{3}(e^{\frac{1}{2}x} + 2)$. Use the Newton-Raphson method, with $f(x) = \frac{1}{3}(e^{\frac{1}{2}x} + 2) - x$ and $x_1 = 1.2$, to find α correct to 2 decimal places. Show all your working. **[4]**

(iv) Given that $x_1 = \ln 36$, explain why the Newton-Raphson method would not converge to a root of $f(x) = 0$. **[2]**

Q6, (Jan 2011, Q5)

The equation

$$x^3 - 5x + 3 = 0 \qquad \text{(A)}$$

may be solved by the Newton-Raphson method. Successive approximations to a root are denoted by $x_1, x_2, \ldots, x_n, \ldots$.

(i) Show that the Newton-Raphson formula can be written in the form $x_{n+1} = \mathrm{F}(x_n)$, where

$$\mathrm{F}(x) = \frac{2x^3 - 3}{3x^2 - 5}.$$ **[3]**

(ii) Find $\mathrm{F}'(x)$ and hence verify that $\mathrm{F}'(\alpha) = 0$, where α is any one of the roots of equation (A). **[3]**

(iii) Use the Newton-Raphson method to find the root of equation (A) which is close to 2. Write down sufficient approximations to find the root correct to 4 decimal places. **[3]**

Q7, (Jan 2013, Q8)

It is required to solve the equation $\ln(x-1) - x + 3 = 0$.

You are given that there are two roots, α and β, where $1.1 < \alpha < 1.2$ and $4.1 < \beta < 4.2$.

(i) The root β can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with $x_1 = 4.15$, find β correct to 3 decimal places. Show all your working. **[2]**

(b) Explain with the aid of a sketch why this iterative formula will not converge to α whatever initial value is taken. **[3]**

(ii) (a) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{3 - 2x_n - (x_n - 1)\ln(x_n - 1)}{2 - x_n}.$$ **[5]**

(b) Use this formula with $x_1 = 1.2$ to find α correct to 3 decimal places. **[3]**

Q9, (Jun 2015, Q6i-iii,v)

It is given that the equation $3x^3+5x^2-x-1=0$ has three roots, one of which is positive.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written

$$x_{n+1}=\frac{6x_n^{\ 3}+5x_n^{\ 2}+1}{9x_n^{\ 2}+10x_n-1}.$$ **[3]**

(ii) A sequence of iterates $x_1, x_2, x_3, \ldots$ which will find the positive root is such that the magnitude of the error in x_2 is greater than the magnitude of the error in x_1. On the graph given in the Printed Answer Book, mark a possible position for x_1. **[1]**

(iii) Apply the iterative formula in part **(i)** when the initial value is $x_1=-1$. Describe the behaviour of the iterative sequence, illustrating your answer on the graph given in the Printed Answer Book. **[2]**

(v) Find the value of the positive root correct to 5 decimal places. **[2]**