The Newton-Raphson Method (From OCR 4726)

Q1, (Jan 2006, Q2)

2	Write as $f(x) = \pm(x - e^{-x})$	B1	Or equivalent
	So $f'(x) = \pm(1 + e^{-x})$	B1	Correct from their f(x)
	Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$	M1	Clear evidence of N-R on their f, f′
	Get $x_1 = 0.56631$, $x_2 = 0.56714$	A1√	At least one to 4d.p.
	Get $x_3 = 0.567(1)$	A1	cao to 3 d.p.

Q2, (Jan 2008, Q5)

(i)	Attempt use of product rule	M1	
	Clearly get $x = 1$	A1	Allow substitution of $x=1$
(ii)	Explain use of tangent for next approx.	B1	Not use of G.C. to show divergence
	Tangents at successive approx. give $x>1$	B1	Relate to crossing x-axis; allow diagram
(iii)	Attempt correct use of N-R with their derivative	M1	
	Get $x_2 = -1$	A1√	
	Get –0.6839, -0.5775, (-0.5672…)	A1	To 3 d.p. minimum
	Continue until correct to 3 d.p.	M1	May be implied
	Get –0.567	A1	cao

Q3, (Jun 2008, Q6i,ii)

(i)	Attempt to use N-R of correct form with clear $f'(x)$ used	M1	
	Get 2.633929, 2.645672	A1	For one correct to minimum of 6 d.p.
		A1√	For other correct from their x_2 in correct NR
		3	
(ii)	$\sqrt{7}$	B1	Allow ±
		1	

Q4, (Jan 2010, Q3i,ii)

(i)	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$	M1	Allow reasonable y-step/x-step
	Equate to gradient of curve at x_1	M1	Allow ±
	Clearly arrive at A.G.	A1	Beware confusing use of ±
	SC Attempt equation of tangent	M1	As $y - f(x_1) = f'(x_1)(x - x_1)$
	Put $(x_2, 0)$ into their equation	M1	
	Clearly arrive at A.G.	A1	
(ii)	Diagram showing at least one more tangent	B1	
	Description of tangent meeting x-axis, used as next starting value	B1	

Q5, (Jun 2010, Q7iii,iv)

(iii) Reasonable attempt to use log/expo. rules M1 Allow derivation either way
Clearly get A.G. A1
Attempt f ′(x) and use at least once in correct N-R formula M1
Get answers that lead to 1.31 A1 Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.

(iv) Show f ′(ln36) = 0 B1
Explain why N-R would not work B1 Tangent parallel to Ox would not meet Ox again or divide by 0 gives an error

Q6, (Jan 2011, Q5)

(i)	$x_{n+1} = x_n - \dfrac{x_n^3 - 5x_n + 3}{3x_n^2 - 5} = \dfrac{2x_n^3 - 3}{3x_n^2 - 5}$	M1 A1 A1	 3	For attempt at N-R formula For correct N-R expression For correct answer (necessary details needed) **AG** Allow omission of suffixes
(ii)	$F'(x) =$ $\dfrac{6x^2(3x^2-5)-6x(2x^3-3)}{(3x^2-5)^2} = \dfrac{6x(x^3-5x+3)}{(3x^2-5)^2}$	M1 M1		For using quotient *OR* product rule to find $F'(x)$ For factorising numerator to show $k(x^3-5x+3)$
	$F'(\alpha) = \dfrac{6\alpha(\alpha^3-5\alpha+3)}{(3\alpha^2-5)^2} = 0$ since $\alpha^3 - 5\alpha + 3 = 0$	A1	3	For correct explanation of **AG**
(iii)	$x_1 = 2 \Rightarrow 1.85714,\ 1.83479,\ 1.83424,\ 1.83424$ $(\alpha =)\ 1.8342$ **SR** For starting value leading to another root allow up to B1 B1 B0	B1 B1 B1	 3	First iterate correct to at least 4 d.p. *OR* $\frac{13}{7}$ For 2 equal iterates to at least 4 d.p. For correct α to 4 d.p. Allow answer rounding to 1.8342 **SR** If not N-R, B0 B0 B0
			9	

Q7, (Jan 2013, Q8)

(i)	(a)	$x_1 = 4.15,\quad x_2 = 4.1474....$ $x_3 = 4.1465....,\quad x_4 = 4.1463....$ $\beta = 4.146$	M1 A1 [2]	Using iterative formula at least once using at least 4dp www	All iterates must be seen
(i)	(b)	Staircase diagram will always move to upper root	B1 B1 B1 [3]	Sketch showing an example $x_1 > \alpha$ Example with $x_1 < \alpha$ Statement Dep on 1st two B	Ignore any statement when $x_1 > \beta$
(ii)	(a)	$\ln(x-1) = x-3 \Rightarrow \ln(x-1)-(x-3)=0$ $\Rightarrow \mathrm{f}(x) = \ln(x-1)-(x-3)$ $\Rightarrow \mathrm{f}'(x) = \dfrac{1}{x-1}-1$ $\Rightarrow x_{n+1} = x_n - \dfrac{\ln(x_n-1)-(x_n-3)}{\dfrac{1}{x_n-1}-1}$ $= x_n - \dfrac{(x_n-1)(\ln(x_n-1)-(x_n-3))}{1-(x_n-1)}$ $= \dfrac{x_n(2-x_n)+(x_n-1)(x_n-3)-(x_n-1)\ln(x_n-1)}{2-x_n}$ $= \dfrac{2x_n - x_n^2 + x_n^2 - 4x_n + 3 - (x_n-1)\ln(x_n-1)}{2-x_n}$ $\Rightarrow x_{n+1} = \dfrac{3-2x_n-(x_n-1)(\ln(x_n-1)}{2-x_n}$	M1 M1 M1 A1 A1 [5]	Get equation in correct form Differentiate Use correct formula Mult by $(x-1)$ **soi**	

(ii)	(b)	1.2 1.152(359) 1.152359 1.158448 1.158448 1.158594 1.158594 1.158594 Root = 1.159	B1 B1 B1 [3]	For x_2 For enough iterates to determine 3dp www	Allow 3 dp x_2 must be right for last B1. Any error is likely to be self-correcting

Q9, (Jun 2015, Q6i-iii,v)

(i)		$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n^3 + 5x_n^2 - x_n - 1}{9x_n^2 + 10x_n - 1}$ $= \frac{x_n(9x_n^2 + 10x_n - 1) - (3x_n^3 + 5x_n^2 - x_n - 1)}{9x_n^2 + 10x_n - 1}$ $= \frac{9x_n^3 + 10x_n^2 - x_n - 3x_n^3 - 5x_n^2 + x_n + 1}{9x_n^2 + 10x_n - 1}$ $= \frac{6x_n^3 + 5x_n^2 + 1}{9x_n^2 + 10x_n - 1}$	B1 M1 A1	Correct derivative seen Combining terms seen as 1 fraction or 2 fractions with common denominator Line above seen ag Must contain suffices.	
			3		
(ii)		A suitable value is shown within range [0.1, 0.25]	B1	The point does not have to be labelled x_1	Accept a tangent which shows this.
			1		
(iii)		$\Rightarrow x_2 = 0 \Rightarrow x_3 = -1$, and statement that values alternate. Clear diagram with tangents from – 1 to 0 and back to –1	B1 B1	Values seen either in words or on graph marked as these values	
(v)		Continuing the above to give root 0.47936	M1 A1	Or any other starting point that converges to the positive root Cao	
			2		