

Small Angle Approximations Exam Questions MS

Q1, (OCR H240/03, Sample Question Paper, Q4)

<p>When <math>\theta</math> is small</p> $1 + \cos \theta - 3 \cos^2 \theta$ $\approx 1 + \left(1 - \frac{1}{2} \theta^2\right) - 3 \left(1 - \frac{1}{2} \theta^2\right)^2$ $= 1 + \left(1 - \frac{1}{2} \theta^2\right) - 3 \left(1 - \theta^2 + \frac{1}{4} \theta^4\right)$ $= 1 + 1 - \frac{1}{2} \theta^2 - 3 + 3 \theta^2 - \frac{3}{4} \theta^4$	<p><b>M1</b></p> <p><b>M1</b></p>	<p><b>1.1a</b></p> <p><b>1.1</b></p>	<p>Attempt to use <math>\cos \theta \approx 1 - \frac{1}{2} \theta^2</math> or  <math>= 1 + \left(1 - \frac{1}{2} \theta^2 + \dots\right) - 3 \left(1 - \frac{1}{2} \theta^2 + \dots\right)^2</math></p> <p>Multiply out</p>	<p><b>OR</b></p> <p><b>M1</b> Attempt to use  <math>\cos \theta \approx 1 - \frac{1}{2} \theta^2</math></p> <p><b>M1</b> use trigonometric identity  <math>1 + \cos \theta - 3 \cos^2 \theta</math>  <math>= 1 + \cos \theta - \frac{3}{2} - \frac{3}{2} \cos 2\theta</math></p>
<p>Since <math>\theta</math> is small, we can neglect the higher order terms</p>	<p><b>E1</b></p>	<p><b>2.5</b></p>	<p>For explanation of loss of <math>\theta^4</math> term and consistent use of notation throughout (Working need not be fully correct)</p>	<p><b>E1</b> For showing clearly which identity has been used and consistent use of notation throughout</p>
<p>so <math>1 + \cos \theta - 3 \cos^2 \theta \approx -1 + \frac{5}{2} \theta^2</math> as required</p>	<p><b>E1</b></p> <p><b>[4]</b></p>	<p><b>2.1</b></p>	<p>AG Clearly obtained www          Condone <math>\theta^4</math> term missing without explanation and inconsistent notation</p>	<p><b>E1</b> AG Clearly obtained www          Condone inconsistent notation</p>

**Q2, (OCR H240/03, Practice Paper Set 1, Q3)**

(i)	$\frac{AC}{\sin \frac{3}{4}\pi} = \frac{1}{\sin(\pi - \frac{3}{4}\pi - \theta)}$ $AC = \frac{\sin \frac{3}{4}\pi}{\sin \frac{1}{4}\pi \cos \theta - \cos \frac{1}{4}\pi \sin \theta}$ $\sin \frac{3}{4}\pi = \sin \frac{1}{4}\pi = \cos \frac{1}{4}\pi \text{ so } AC = \frac{1}{\cos \theta - \sin \theta}$	<p>M1</p> <p>M1</p> <p>E1</p> <p>[3]</p>	<p>2.1</p> <p>2.1</p> <p>2.2a</p>	<p>Attempt sine rule</p> <p>For expanding <math>\sin(\frac{1}{4}\pi - \theta)</math></p> <p>AG, so must show sufficient working; e.g. stating <math>\sin \frac{3}{4}\pi = \sin \frac{1}{4}\pi = \cos \frac{1}{4}\pi</math> or using <math>\frac{1}{\sqrt{2}}</math> oe for each</p>	
(ii)	$AC = \left(1 + (-\theta - \frac{1}{2}\theta^2)\right)^{-1}$ $AC = 1 + (-1)(-\theta - \frac{1}{2}\theta^2) + \frac{(-1)(-2)}{2}(-\theta - \frac{1}{2}\theta^2)^2 + \dots$ $AC \approx 1 + \theta + \frac{3}{2}\theta^2$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p>	<p>Using both small angle approximations</p> <p>Attempt binomial expansion of <math>AC</math>, with at least the first two terms present</p> <p><math>p = 1</math></p> <p><math>q = \frac{3}{2}</math></p>	

**Q3, (OCR H240/02, Practice Paper Set 3, Q3)**

$$\frac{1 - \frac{1}{8}\theta^2}{1 + \theta} = 0.825$$

$$0.125\theta^2 + 0.825\theta - 0.175 = 0$$

$$\theta = 0.206 \text{ or } -6.81 \text{ (3 sf)}$$

$$\text{Discard } -6.81 \text{ as not small. } \theta = 0.206 \text{ (3 sf)}$$

M1

1.1a

A1

1.1

A1

1.1

A1

2.3

[4]

BC

Statement needed and  $\theta = 0.206$  alone