

Partial Fractions Exam Questions MS (From Legacy OCR C4)

Q1 (Jun 2005, Q8) [Modified]

(i) $3x+4 \equiv A(2+x)^2+B(2+x)(1+x) + C(1+x)$	M1	Accept \equiv or $=$ If identity used, award 'A' mark, if cover-up rule used, award 'B' mark. <u>Any</u> correct eqn for B from identity
$A = 1$	A/B1	
$C = 2$	A/B1	
$A+B=0$ or $4A+3B+C=3$ or $4A+2B+C=4$	A1	
$B = -1$	A1 5	
	B1	

Q2 (Jan 2006, Q7) [Modified]

(i) $A = 3$	B1	For correct value stated
$C = 1$	B1	For correct value stated
$11 + 8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$	M1	AEF; any suitable identity
e.g. $A - B = 0, 2A + B - C = 8, A + 2B + 2C = 11$	A1	For any correct (f.t.) equation involving B
$B = 3$	A1	5

Q3 (Jun 2007, Q1) [Modified]

(i) Correct format $\frac{A}{x+2} + \frac{B}{x-3}$	M1	s.o.i. in answer
$A = 1$ and $B = 2$	A1 2	

Q4 (Jan 2008, Q1)

(i) Correct format $\frac{A}{x+1} + \frac{B}{x+2}$	M1	stated or implied by answer
$-\frac{1}{x+1}$ or $A = -1$	A1	
$+\frac{2}{x+2}$ or $B = 2$	A1	
	3	

Q5 (Jun 2010, Q3)

$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$	M1	For correct format
$A(x-1)(x-2) + B(x-2) + C(x-1)^2 \equiv x^2$	M1	
$A = -3$	A1	
$B = -1$	A1	(B1 if cover-up rule used)
$C = 4$	A1	(B1 if cover-up rule used)

Q6 (Jun 2013, Q1)

$$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

[If no partial fractions seen anywhere, B0]

$$(x-7)(x-2) \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$$

[Allow careless minor error but not algebraic method error]

or any equiv identity such as

$$\frac{(x-7)(x-2)}{(x-1)^2} \equiv A + \frac{B(x+2)}{(x-1)} + \frac{C(x+2)}{(x-1)^2} \text{ (or even the}$$

identity on the 1st line), in which values of x are substituted (or cfs compared)

$$A = 4, B = -3, C = 2 \text{ or } \frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2} \text{ ISW}$$

The 3 @ A1 are dep on the used identity being correct.

Cover-up: $A=4, C=2$ score B1,B1; $B = -3$ needs M1, then A1

B1

M1

A1,1,1

[5]

SC
$$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2} + \frac{Bx+C}{(x-1)^2}$$

[If no partial fractions seen anywhere, B0]

$$(x-7)(x-2) \equiv A(x-1)^2 + (Bx+C)(x+2)$$

[Allow careless minor error but not algebraic method error]

or any equivalent identity (as in previous column) (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)

$$A = 4, B = -3, C = 5 \text{ or } \frac{4}{x+2} + \frac{-3x+5}{(x-1)^2}$$

Q7, (Edexcel 6666, Jan 2013, Q3)

$$\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv 3 + \frac{5x-4}{(x+2)(3x-1)}$$

$$\text{So, } \frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$$

$$5x - 4 \equiv B(3x-1) + C(x+2)$$

Either $x: 5 = 3B + C$, constant: $-4 = -B + 2C$

or

$$x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$$

$$x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$$

$$\text{So, } \frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$$

their constant term = 3 B1

Forming a correct identity. B1

Attempts to find the value of either one of their B or their C from their identity. M1

Correct values for their B and their C , which are found using $5x - 4 \equiv B(3x-1) + C(x+2)$ A1

[4]

Q8, (Edexcel IAL, C34, Jun 2016, Q4)

<p>(a)</p>	$\begin{array}{r} x^2 + x - 12 \overline{) x^4 + x^3 - 7x^2 + 8x - 48} \\ \underline{x^4 + x^3 - 12x^2} \\ 5x^2 + 8x - 48 \\ \underline{5x^2 + 5x - 60} \\ 3x + 12 \end{array}$ <p>M1: Divides $x^4 + x^3 - 7x^2 + 8x - 48$ by $x^2 + x - 12$ to get a quadratic quotient and a remainder of the form $\alpha x + \beta$ where α and β are not both zero A1: Correct quotient and remainder</p>	<p>M1A1</p>
	$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + 5 + \frac{3(x+4)}{(x+4)(x-3)}$ <p>Writes their answer as</p> $\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv \text{Their Quotient} + \frac{\text{Their Remainder}}{(x+4)(x-3)}$	<p>M1</p>
	$\equiv x^2 + 5 + \frac{3}{(x-3)}$ <p>or states $A = 5, B = 3$</p>	<p>A1</p>
<p>(4)</p>		

<p>(b)</p>	$g'(x) = 2x - \frac{3}{(x-3)^2}$	<p>M1: $x^2 + A + \frac{B}{x-3} \rightarrow 2x \pm \frac{B}{(x-3)^2}$ A1: $x^2 + A + \frac{B}{x-3} \rightarrow 2x - \frac{B}{(x-3)^2}$ Follow through their B or the letter B or a made up B.</p>	<p>M1A1ft</p>
<p>Special Case: If they write $g(x)$ as $x^2 + 5 + \frac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x +$ the quotient rule on $\frac{3x+12}{(x-3)}$ then the M mark is available but not the A1ft. It must be the correct quotient rule and the numerator must be a linear expression.</p>			
$g'(4) = 2 \times 4 - \frac{3}{(4-3)^2} (=5)$		<p>Substitutes $x = 4$ into their derivative</p>	<p>M1</p>
<p>Uses $m = g'(4) = (5)$ with $(4, g(4)) = (4, 24)$ to form eqn of tangent</p>			
$y - 24 = 5(x - 4)$		<p>Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4, g(4))$</p>	<p>M1</p>
$y = 5x + 4$		<p>Cso. This mark may be withheld for an incorrect "A" earlier or any incorrect work leading to a correct gradient.</p>	<p>A1</p>
			<p>(5)</p>

<p>(a)</p>	$ \begin{array}{r} x^2 + x - 12 \overline{) x^4 + x^3 - 7x^2 + 8x - 48} \\ \underline{x^4 + x^3 - 12x^2} \\ 5x^2 + 8x - 48 \\ \underline{5x^2 + 5x - 60} \\ 3x + 12 \end{array} $ <p>M1: Divides $x^4 + x^3 - 7x^2 + 8x - 48$ by $x^2 + x - 12$ to get a quadratic quotient and a remainder of the form $\alpha x + \beta$ where α and β are not both zero A1: Correct quotient and remainder</p>	<p>M1A1</p>
	$ \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + 5 + \frac{3(x+4) \text{ or } 3x+12}{(x+4)(x-3)} $ <p>Writes their answer as</p> $ \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv \text{Their Quotient} + \frac{\text{Their Remainder}}{(x+4)(x-3)} $ $ \equiv x^2 + 5 + \frac{3}{(x-3)} \text{ or states } A = 5, B = 3 $	<p>M1</p> <p>A1</p> <p style="text-align: right;">(4)</p>

<p>(b)</p>	$g'(x) = 2x - \frac{3}{(x-3)^2}$	<p>M1: $x^2 + A + \frac{B}{x-3} \rightarrow 2x \pm \frac{B}{(x-3)^2}$ A1: $x^2 + A + \frac{B}{x-3} \rightarrow 2x - \frac{B}{(x-3)^2}$ Follow through their B or the letter B or a made up B.</p>	<p>M1A1ft</p>
	<p style="text-align: center;">Special Case: If they write $g(x)$ as $x^2 + 5 + \frac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x +$ the quotient rule on $\frac{3x+12}{(x-3)}$ then the M mark is available but not the A1ft. It must be the correct quotient rule and the numerator must be a linear expression.</p>		
	$g'(4) = 2 \times 4 - \frac{3}{(4-3)^2} (= 5)$	<p>Substitutes $x = 4$ into their derivative</p>	<p>M1</p>
	<p>Uses $m = g'(4) = (5)$ with $(4, g(4)) = (4, 24)$ to form eqn of tangent</p>		
	$y - 24 = 5(x - 4)$	<p>Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4, g(4))$</p>	<p>M1</p>
	$y = 5x + 4$	<p>Cso. This mark may be withheld for an incorrect "A" earlier or any incorrect work leading to a correct gradient.</p>	<p>A1</p>
	<p style="text-align: right;">(5)</p> <p style="text-align: right;">(9 marks)</p>		