Parametric Equations (From OCR 4724)

Q1, (Jun 2005, Q7)

(i) $dy / dx = (dy/dt) / (dx/dt)$ $= (-1/t^2) / 2t$ as unsimplified expression	M1 A1	(S.R.Award M1 for attempt to change to cartesian eqn & differentiate + A1 for dy/dx or dx/dy in terms of x or y)
$= -1 / 2t^3$ as simplified expression	A1 **3**	Not $1/-2t^3$. Not in terms of x &/or y.
(ii) $(4,-1/2) \rightarrow t = -2$ <u>only</u> Satis attempt to find equation of tgt $x - 16y = 12$ <u>only</u>	B1 M1 A1 **3**	Using $t = -2$ or 2 **AG**
(iii)	M1	For substituting $(t^2, 1/t)$ into tgt eqn <u>or</u> solving simult tgt & their cartes eqns
$t^3 - 12t - 16 = 0$ <u>or</u> $16y^3 + 12y^2 - 1 = 0$ <u>or</u> $x^3 - 24x^2 + 144x - 256 = 0$	A1	For simplified equiv non-fract cubic
$t = 4$ (only) ISW giving cartesian coords	B2 **4**	S.R. Award B1 for "4 or –2". S.R. If B0, award M1 for clear indic of method of soln of correct eqn. **10**

Q2, (Jan 2009, Q6)

(i) Solve $0 = t - 3$ & subst into $x = t^2 - 6t + 4$ — M1

Obtain $x = -5$ — A1 **(2)** — $(-5,0)$ need not be quoted

N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)

(ii) Attempt to eliminate t — M1

Simplify to $x = y^2 - 5$ ISW — A1 **(2)**

(iii) Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form — M1 — Award anywhere in Que

Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-)$\frac{1}{2}(x+5)^{-\frac{1}{2}}$ — A1

If $t = 2$, $x = -4$ and $y = -1$ — B1 — Awarded anywhere in **(iii)**

Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn — M1

$x + 2y + 6 = 0$ AEF(without fractions) ISW — A1 **(5)**

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Q3, (Jun 2009, Q5)

(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used	M1
	$= \frac{4t+3t^2}{2+2t}$	A1
	Attempt to find t from one/both equations	M1 <u>or</u> diff (ii) cartesian eqn $\to$ M1
	State/imply $t=-3$ is only solution of both equations	A1 subst $(3,-9)$, solve for $\frac{dy}{dx} \to$ M1
	Gradient of curve $= -\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$	A1 **5** grad of curve $= -\frac{15}{4} \to$ A1

[**SR** If $t=1$ is given as solution & not disqualified, award A0 + √A1 for grad $= -\frac{15}{4}$ & $\frac{7}{4}$;

If $t=1$ is given/used as only solution, award A0 + √A1 for grad $= \frac{7}{4}$]

(ii)	$\frac{y}{x} = t$	B1
	Substitute into either parametric eqn	M1
	Final answer $x^3 = 2xy + y^2$	A2 **4**

[**SR** Any correct unsimplified form (involving fractions or common factors) $\to$ A1]

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Q4, (Jun 2010, Q7)

(i)	Differentiate x as a quotient, $\frac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\frac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$	M1	or product clearly defined
	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$	A1	WWW → 2
	$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$	B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1	quoted/implied and used
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks)	*A1	ignore ref $t=-1, t=-3$
	State <u>squares</u> +ve or $(t+1)^2$ & $(t+3)^2$ + ve $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}$ +ve	dep*A1 **6**	or $\left(\frac{t+1}{t+3}\right)^2$ + ve. Ignore ≥ 0
(ii)	Attempt to obtain t from either the x or y equation	M1	No accuracy required
	$t = \frac{2-x}{x-1}$ AEF <u>or</u> $t = \frac{2}{y} - 3$ AEF	A1	
	Substitute in the equation not yet used in this part	M1	or equate the 2 values of t
	Use correct meth to eliminate ('double-decker') fractions	M1	
	Obtain <u>$2x + y = 2xy + 2$</u> ISW AEF	A1 **5**	but not involving fractions **11**

Q5, (Jan 2011, Q4)

(i)	Attempt to use $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x}$	M1	Not just quote formula
	$\frac{4}{2t}$ or $\frac{2}{t}$	A1 **2**	
(ii)	Subst $t=4$ into their (i), invert & change sign	M1	
	Subst $t=4$ into (x,y) & use num grad for tgt/normal	M1	
	$y = -2x + 52$ AEF CAO (no f.t.)	A1 **3**	Only the eqn of normal accepted
(iii)	Attempt to eliminate t from the 2 given equations	M1	
	$x = 2 + \frac{y^2}{16}$ or $y^2 = 16(x-2)$ AEF ISW	A1 **2**	Mark at earliest acceptable form.
		7	

Q6, (Jun 2011, Q8)

Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded

(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$		
	OR sub $t = -2$ into para eqs, obtain $(-1,-3)$ & state $y = 3x$		
	OR other similar methods producing (or verifying) $t = -2$	B1	
	Value of t at other point is 2	B1 **2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	
	$= -(t+1)^2$	A1	or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1	
	Gradient normal = 1 cao	A1	
	Subst $t = -2$ into the parametric eqns.	M1	to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal WWW	A1 **6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1	
	Produce $t = 0$ as final answer cao	A1 **2**	This is dep on final A1 in (ii)
	N.B. If $y = x - 2$ is found fortuitously in (ii) (& ∴ given A0 in (ii)), you must award A0 here in (iii).		
(iv)	Attempt to eliminate t from the parametric equations	M1	
	Produce any correct equation	A1	e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1 **3**	Must be seen in (iv)
	{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}		

Q7, (Jan 2013, Q5)

(i)		their $\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1		
		$\dfrac{dy}{dx}=\dfrac{2\sin\theta}{3\cos\theta}$	A1		
		their $\dfrac{dy}{dx}=\dfrac{1}{2}$	M1		
		$\tan\theta=\dfrac{3}{4}$	A1	If $\tan\theta=\dfrac{3}{4}$ not seen, award this A1 only if coords are correct	
		$(3.8,-0.6)$ or $\left(\dfrac{19}{5},-\dfrac{3}{5}\right)$ or $x=3.8,\ y=-0.6$	A1 **[5]**		
(ii)		Manipulating equations into form $\sin\theta=f(x)$ and $\cos\theta=g(y)$ and then using $\sin^2\theta+\cos^2\theta=1$	M1	<u>If part (ii) is attempted first, and then part (i), allow</u> B1 for obtaining $\dfrac{dy}{dx}=\dfrac{4(x-2)}{9(y-1)}$ M1 for equating their $\dfrac{dy}{dx}$ to $\dfrac{1}{2}$ A1 for obtaining $9y-8x=-7$ M1 for eliminating x or y from above eqn... A1 for $(3.8,-0.6)$	<u>the following marks in part (i):-</u> ....and their Cartesian equation
		$\dfrac{(x-2)^2}{9}+\dfrac{(1-y)^2}{4}=1$ oe www ISW Accept e.g. $\left(\dfrac{x-2}{3}\right)^2$ $4x^2+9y^2-16x-18y-11=0$	A1 **[2]**		

Q8, (Jun 2013, Q9)

(i)	$\frac{dy}{dt} = 2(+) - \frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW	B1, B1		
	$\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right), \frac{2t^3-2}{-t}, -t^2\left(2 - \frac{2}{t^3}\right)$ oe	B1 **[3]**	ISW. Must not involve (implied)'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2-2t^{-3}}{-t^2}$
(ii)	(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$	M1		
	$t = 1 \rightarrow$ (stationary point) = (0, 3)	A1	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$	
	Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$	M1		
	Hence (0, 3) is a minimum point www	A1 **[4]**	Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
(iii)	Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y	M1		
	$y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	A1 **[2]**		

Q9, (Jun 2014, Q7)

(i)	$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of $x = 2\sin t$
	$\frac{dy}{dx} = \text{their} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
	$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
	$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to $1 - 2\sin t$ www	A1	or equivalent intermediate step	
	$(1, 1\frac{1}{2})$	B1 **[5]**	NB $t = \frac{\pi}{6}$	from $1 - 2\sin t = 0$
(ii)	$(y =)\ 1 - 2\sin^2 t + 2\sin t$	B1	may be awarded after correct substitution for x eg $(y =)\ 1 - {}^{x^2}/_4 - \sin^2 t + 2\sin t$	or $(y =)\ x + \cos 2t$
	substitution of $\sin t = \frac{1}{2}x$ to eliminate t	M1		substitution of $t = \sin^{-1}({}^x/_2)$ to eliminate t
	$y = 1 + x - \frac{1}{2}x^2$ oe isw	A1 **[3]**	or B3 www	$y = x + \cos 2(\sin^{-1}({}^x/_2))$ oe isw

(iii)	$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \le 2$ or $\lvert x \rvert \le 2$	B1	cao	
	sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	
	positive y-intercept and one distinguishing feature isw	A1		one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), y – intercept is (0, 1), x-intercept is $(1-\sqrt{3}, 0)$
		[3]		

Q10, (Jun 2015, Q10)

(i)	$\frac{A}{x}+\frac{B}{x+2}$	**B1**		award if only implied by answer
	$x+8=A(x+2)+Bx$ soi	**M1**	allow one sign error	clearing fractions successfully
	$A=4$ *and* $B=-3$	**A1**		if **M0**, **B1** for each value www
		[3]		
(ii)	quotient (P) is 7	**B1**		
	$2x+16$ seen	**B1**	if **B0**, **B1** for Q =8 and **B1** for R = – 6 www	eg as remainder or in division chunking
	$7+\frac{8}{x}-\frac{6}{x+2}$	**B1**		or allow P = 7, Q = 8 R = – 6
		[3]		

(iii)	$t = \mathrm{f}(x)$	**M1***	from $x = \dfrac{2t}{1-t}$; **M0** for $t = \mathrm{g}(y)$	
	$t = \dfrac{x}{x+2}$	**A1**	or **B2** if unsupported	
	$y = 3 \times \text{their } \dfrac{x}{x+2} + \dfrac{4}{\text{their } \dfrac{x}{x+2}}$	**M1dep***		
	eg $\dfrac{3x^2 + (8+4x)(x+2)}{x(x+2)}$ and completion to			at least one correct, constructive, intermediate step shown
	$y = \dfrac{7x^2 + 16x + 16}{x(x+2)}$ www **AG**	**A1**		if **M0M0**, **SC2** for substitution of $x = \dfrac{2t}{1-t}$ in RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \dfrac{4}{t}$ www
		[4]		
(iv)	$\int \text{their } (P + \dfrac{Q}{x} + \dfrac{R}{x+2})[\mathrm{d}x]$	**M1***	where P, Q and R are constants obtained in (ii)	allow omission of dx
	$\mathrm{F}[x] = 7x + 8\ln x - 6\ln(x+2)$	**A1FT**	allow recovery from omission of brackets in subsequent working	if **M0**, **SC1** for $Px + Q\ln x + R\ln(x+2)$ where constants are unspecified or arbitrary
	$\mathrm{F}[2] - \mathrm{F}[1]$	**M1dep***		
	$7 - 4\ln 2 + 6\ln 3$	**A1** **[4]**		

Q11. (Jun 2016, Q9)

(i)	$\sin t \sin 2t = 0$ oe seen	**M1**		**NB** $t = 0, \frac{1}{2}\pi, \pi$
	(0, 0) (1, 0) and (2, 0) or $x = 0, x = 1, x = 2$ cao	**A2**	**A1** for two of three correct	deduct 1 mark if all three correct plus extra values
		[3]		if **A0**, allow **SC1** for $t = 0, \frac{1}{2}\pi, \pi$ if unsupported, full marks for all three values correct
(ii)	$\left[\frac{dy}{dt}\right] = 2\sin t\cos 2t + \cos t\sin 2t$	**B1**	or $4\sin t\cos^2 t - 2\sin^3 t$	
	$\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ or $\frac{(4\sin t\cos^2 t - 2\sin^3 t)}{\sin t}$	**M1**	allow sign errors and/or one incorrect coefficient	
	substitution of $\sin 2t = 2\sin t\cos t$ in their	**M1**	may be seen before differentiation	
	$\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ and completion to $2\cos 2t + 2\cos^2 t$ www **NB AG**	**A1**	at least one correct intermediate step needed	
	eg $2(2\cos^2 t - 1) + 2\cos^2 t = 0$ or $2\cos 2t + 2\times\frac{1}{2}(1 + \cos 2t) = 0$	**M1**	use of double angle formula to obtain quadratic equation in eg $\cos t$ or linear equation in $\cos 2t$; may be seen before differentiation	mark intent: allow sign error, bracket error, omission of one coefficient
	$(1+\frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}})$ oe isw	**A1**		eg $(\frac{\sqrt{3}+3}{3}, -\frac{4\sqrt{3}}{9})$
	$(1-\frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}})$ oe isw	**A1** **[7]**	if **A0**, **A0**, allow **A1** for both x values correct	

(iii)	$y = 2(1 - \cos^2 t)\cos t$ oe may be implicit equation, may be implied by partial substitution for $\cos t$ eg $(1-x)^2 + \dfrac{y}{2\cos t} = 1$	**M1**	or $\dfrac{dy}{dx} = 6\cos^2 t - 2$	use of double angle formula (and Pythagoras) to obtain expression for y or $\dfrac{dy}{dx}$ in terms of $\cos t$ only;
	$y = 2(1-(1-x)^2)(1-x)$	**M1**	or $\dfrac{dy}{dx} = 6(1-x)^2 - 2$	substitution of $\cos t = \pm 1 \pm x$ to obtain expression in terms of x only allow sign errors, bracket errors or minor slips in arithmetic eg omission of 2 for these method marks
	$y = 2x^3 - 6x^2 + 4x$ or $y = 2x(x^2 - 3x + 2)$ or $y = 2x(x-1)(x-2)$ oe cao	**A1** **[3]**	integration and substitution of eg (0, 0) to obtain correct answer must see $y =$ at some stage for **A1**	
(iv)	cubic with two turning points and of correct orientation through (0, 0)	**M1**		
	x-intercepts correct and only for $0 \le x \le 2$	**A1** **[2]**		