Mixed Sequences Exam Questions (from OCR 4722)

Jun 2006

6 (i) John aims to pay a certain amount of money each month into a pension fund. He plans to pay £100 in the first month, and then to increase the amount paid by £5 each month, i.e. paying £105 in the second month, £110 in the third month, etc.

If John continues making payments according to this plan for 240 months, calculate

(a) how much he will pay in the final month, [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

Jun 2007

7 (a) In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is –4 and the sum to infinity is 9. Find the common ratio. [7]

Jun 2008

10 Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

Jun 2010

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

(i) State the values of u_1, u_2 and u_3. **[1]**

(ii) Evaluate $\sum_{n=1}^{40} u_n$. **[3]**

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of p such that $u_p = w_3$. **[3]**

Jun 2010

9 A geometric progression has first term a and common ratio r, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. **[3]**

(ii) Given that the geometric progression converges, find the exact value of r. **[5]**

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer a. **[4]**

Jan 2012

6 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 85 - 5n$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

(ii) Find $\sum_{n=1}^{20} u_n$. **[3]**

(iii) Given that u_1, u_5 and u_p are, respectively, the first, second and third terms of a geometric progression, find the value of p. **[4]**

(iv) Find the sum to infinity of the geometric progression in part **(iii)**. **[2]**

Jan 2013

6 **(i)** The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x. **[3]**

(ii) The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

(a) Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case. **[4]**

(b) Find the other possible value of x that also gives a geometric progression. **[4]**

Jun 2013

6 Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses 6 g of the chemical and in the second experiment she uses 7.8 g of the chemical.

(i) Given that the amounts of the chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments. **[3]**

(ii) Instead it is given that the amounts of the chemical used form a geometric progression. Sarah has a total of 1800 g of the chemical available. Show that N, the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leqslant 91,$$

and use logarithms to calculate the value of N. **[6]**
