Mixed Sequences Exam Questions MS (from OCR 4722)

Jun 2006, Q6

(i)	(a) $100+239\times5=£1295$	M1 A1	 2	For relevant use of $a + (n - 1)d$ For correct value 1295
	(b) $\frac{1}{2}\times240\times(100+1295)=£167400$	M1 A1	 2	For relevant use of $\frac{1}{2}n(a+l)$ or equivalent For correct value 167400
(ii)	$100r^{239}=1500\Rightarrow r=1.01139...$ Hence total is $\dfrac{100(1.01139^{240}-1)}{1.01139-1}=£124359$	B1 M1 A1 M1 A1	 5	For correct statement of $100r^{239}=1500$ Attempt to find r For correct value 1.01 For relevant use of GP sum formula For correct value 124359 (3 s.f. or better)
			9	

Jun 2007, Q7

(a)	$S_{70}=\frac{70}{2}\{(2\times12)+(70-1)d\}$	M1		Attempt S_{70}
		A1		Obtain correct unsimplified expression
	$35(24+69d)=12915$	M1		Equate attempt at S_{70} to 12915, and attempt to find d
	$d=5$	A1	4	Obtain $d=5$
R				
	$\frac{70}{2}\{12+l\}=12915$	M1		Attempt to find d by first equating $^{n}/_{2}(a+l)$ to 12915
	$l=357$	A1		Obtain $l=357$
	$12+69d=357$	M1		Equate u_{70} to l
	$d=5$	A1		Obtain $d=5$
(b)	$ar=-4$	B1		Correct statement for second term
	$\frac{a}{1-r}=9$	B1		Correct statement for sum to infinity
	$\frac{-4}{r}=9-9r$ or $a=9-\left(9\times\frac{-4}{a}\right)$	M1		Attempt to eliminate either a or r
	$9r^2-9r-4=0$ $\quad a^2-9a-36=0$	A1		Obtain correct equation (no algebraic denominators/brackets)
	$(3r-4)(3r+1)=0$ $\quad (a+3)(a-12)=0$	M1		Attempt solution of three term quadratic equation
	$r=\frac{4}{3}$, $r=-\frac{1}{3}$ $\quad a=-3$, $a=12$	A1		Obtain at least $r=-\frac{1}{3}$ (from correct working only)
	Hence $r=-\frac{1}{3}$	A1	7	Obtain $r=-\frac{1}{3}$ only (from correct working only) SR: answer only / T&I is B2 only
			11	

Jun 2008, Q10

(i)	$u_{15} = 2 + 14 \times 0.5$ $= 9$ km	**M1** **A1** 2	Attempt use of $a + (n-1)d$ Obtain 9 km
(ii)	$u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$	**B1** **M1** **A1**	State, or imply, $r = 1.1$ Attempt u_{20}, using ar^{n-1} Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
	OR		
		B1 **M1** **A1** 3	State, or imply, $r = 1.1$ Attempt to solve $ar^{n-1} = 12$ Obtain $n = 20$ (allow $n \geq 20$)
(iii)	$\frac{2(1.1^n - 1)}{(1.1-1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	**B1** **M1** **A1** **A1** 4	State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1-1)}$ Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve Obtain 26, or 25.2 or better Conclude $n = 26$ only, or equiv eg Day 26
(iv)	swum $= 2 \times 30 = 60$ km run $= \frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$ $= 277.5$ km cycle $= \frac{2(1.1^{30} - 1)}{(1.1-1)}$ $= 329.0$ km total $= 666$ km	**B1** **M1** **M1** **A1** 4	Obtain 60 km, or 2 x 30km Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$ Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$ Obtain 666 or 667 km

Jun 2010 , Q4

4 (i)	$u_1 = 6$, $u_2 = 11$, $u_3 = 16$	**B1**	**1**	State 6, 11, 16
(ii)	$S_{40} = {}^{40}/_{2}\,(2 \times 6 + 39 \times 5)$ $= 4140$	**M1** **M1** **A1**	 **3**	Show intention to sum the first 40 terms of a sequence Attempt sum of their AP from (i), with $n = 40$, $a =$ their u_1 and $d =$ their $u_2 - u_1$ Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p-1) \times 5 = 56$ $p = 11$	**B1** **M1** **A1** 7	 **3**	State or imply $w_3 = 56$ Attempt to solve $u_p = k$ Obtain $p = 11$

Jun 2010, Q9

(i) $ar = a + d, \quad ar^3 = a + 2d$
$2ar - ar^3 = a$
$ar^3 - 2ar + a = 0$
$r^3 - 2r + 1 = 0$ **A.G.**

M1 Attempt to link terms of AP and GP, implicitly or explicitly.

M1 Attempt to eliminate d, implicitly or explicitly, to show given equation.

A1 **3** Show $r^3 - 2r + 1 = 0$ convincingly

(ii) $f(r) = (r-1)(r^2 + r - 1)$

$r = \dfrac{-1 \pm \sqrt{5}}{2}$

Hence $r = \dfrac{-1+\sqrt{5}}{2}$

B1 Identify $(r-1)$ as factor or $r = 1$ as root

M1* Attempt to find quadratic factor

A1 Obtain $r^2 + r - 1$

M1d* Attempt to solve quadratic

A1 **5** Obtain $r = \dfrac{-1+\sqrt{5}}{2}$ only

(iii) $\dfrac{a}{1-r} = 3 + \sqrt{5}$

$a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$

$a = {}^9/_2 - {}^5/_2$
$a = 2$

M1 Equate S_∞ to $3 + \sqrt{5}$

A1 Obtain $\dfrac{a}{1-\left(\frac{-1+\sqrt{5}}{2}\right)} = 3 + \sqrt{5}$

M1 Attempt to find a

A1 **4** Obtain $a = 2$

12

Jan 2012, Q6

(i)		$u_1 = 80$	**B1**	State 80
		$u_2 = 75, u_3 = 70$	**B1** **[2]**	State 75 and 70
(ii)		$S_{20} = \frac{20}{2}(2 \times 80 + 19 \times -5)$ $= 650$	**M1**	Show intention to sum 1st 20 terms of an arithmetic sequence
			M1	Attempt use of correct sum formula for an AP, with $n = 20$, $a = 80$, $d = \pm 5$
			A1 **[3]**	Obtain 650

Jan 2013, Q6

(i)		$(x+4)-2x=(2x-7)-(x+4)$ **OR** $2x+d=x+4 \quad 2x+2d=2x-7$ $2x=15$ $x=7.5$	M1 A1 A1 **[3]**	Attempt to eliminate d to obtain equation in x only Obtain correct equation in just x Obtain $x=7.5$
(ii)	**(a)**	terms are 16, 12, 9 $^{12}/_{16}=0.75$, $^{9}/_{12}=0.75$ common ratio of 0.75 so GP $S_\infty = {}^{16}/_{1-0.75}$ $=64$	B1 B1 M1 A1 **[4]**	List 3 terms Convincing explanation of why it is a GP Attempt use of $^{a}/_{1-r}$ Obtain 64

Jun 2013

(i)		$S_{30} = {}^{30}/_{2}\,(2 \times 6 + 29 \times 1.8)$	M1	Use $d = 1.8$ in AP formula
			A1	Correct unsimplified S_{30}
		$= 963$	A1 **[3]**	Obtain 963
(ii)		$r = {}^{7.8}/_{6} = 1.3$	M1	Use $r = 1.3$ in GP formula
		$\frac{6(1-1.3^N)}{1-1.3} \leq 1800$	A1	Correct unsimplified S_N
		$1 - 1.3^N \geq -90$	M1	Link sum of GP to 1800 and attempt to rearrange to $1.3^N \leq k$
		$1.3^N \leq 91$ **AG**	A1	Obtain given inequality