

Mixed Sequences Exam Questions MS (from OCR 4722)

Jun 2006, Q6

(i)	(a) $100 + 239 \times 5 = \text{£}1295$	M1 A1	2	For relevant use of $a + (n - 1)d$ For correct value 1295
	(b) $\frac{1}{2} \times 240 \times (100 + 1295) = \text{£}167400$	M1 A1		For relevant use of $\frac{1}{2}n(a + l)$ or equivalent For correct value 167400
(ii)	$100r^{239} = 1500 \Rightarrow r = 1.01139\dots$	B1 M1 A1 M1 A1	5	For correct statement of $100r^{239} = 1500$ Attempt to find $r$ For correct value 1.01 For relevant use of GP sum formula For correct value 124359 (3 s.f. or better)
	Hence total is $\frac{100(1.01139^{240} - 1)}{1.01139 - 1} = \text{£}124359$			
				9

Jun 2007, Q7

R	(a) $S_{70} = \frac{70}{2} \{ (2 \times 12) + (70 - 1)d \}$	M1	4	Attempt $S_{70}$
	$35(24 + 69d) = 12915$	A1 M1		Obtain correct unsimplified expression Equate attempt at $S_{70}$ to 12915, and attempt to find $d$
	$d = 5$	A1		Obtain $d = 5$
	$\frac{70}{2} \{ 12 + l \} = 12915$	M1		Attempt to find $d$ by first equating $\frac{n}{2}(a + l)$ to 12915
	$l = 357$	A1		Obtain $l = 357$
	$12 + 69d = 357$	M1		Equate $u_{70}$ to $l$
	$d = 5$	A1		Obtain $d = 5$
	(b) $ar = -4$	B1		Correct statement for second term
	$\frac{a}{1-r} = 9$	B1		Correct statement for sum to infinity
	$\frac{-4}{r} = 9 - 9r$ or $a = 9 - (9 \times \frac{-4}{a})$	M1		Attempt to eliminate either $a$ or $r$
$9r^2 - 9r - 4 = 0$ $a^2 - 9a - 36 = 0$	A1	Obtain correct equation (no algebraic denominators/brackets)		
$(3r - 4)(3r + 1) = 0$ $(a + 3)(a - 12) = 0$	M1	Attempt solution of three term quadratic equation		
$r = \frac{4}{3}, r = -\frac{1}{3}$ $a = -3, a = 12$	A1	Obtain at least $r = -\frac{1}{3}$ (from correct working only)		
Hence $r = -\frac{1}{3}$	A1	7	Obtain $r = -\frac{1}{3}$ only (from correct working only) SR: answer only / T&I is B2 only	
			<b>11</b>	

**Jun 2008, Q10**

(i)  $u_{15} = 2 + 14 \times 0.5$   
 $= 9 \text{ km}$

**M1** Attempt use of  $a + (n - 1)d$   
**A1** Obtain 9 km

**2**

(ii)  $u_{20} = 2 \times 1.1^{19} = 12.2$

**B1** State, or imply,  $r = 1.1$

$u_{19} = 2 \times 1.1^{18} = 11.1$

**M1** Attempt  $u_{20}$ , using  $ar^{n-1}$

**A1** Obtain  $u_{20} = 12.2$ , and obtain  $u_{19} = 11.1$

**OR**

**B1** State, or imply,  $r = 1.1$

**M1** Attempt to solve  $ar^{n-1} = 12$

**A1** Obtain  $n = 20$  (allow  $n \geq 20$ )

**3**

(iii)  $\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$

**B1** State or imply  $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$

$1.1^n > 11$

**M1** Link (any sign) their attempt at  $S_N$  (of a GP) to 200 and attempt to solve

$n > \frac{\log 11}{\log 1.1}$

**A1** Obtain 26, or 25.2 or better

$n > 25.2$  ie Day 26

**A1** Conclude  $n = 26$  only, or equiv eg Day 26

**4**

(iv) swum =  $2 \times 30 = 60 \text{ km}$

**B1** Obtain 60 km, or  $2 \times 30\text{km}$

run =  $\frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$   
 $= 277.5 \text{ km}$

**M1** Attempt sum of AP,  $d = 0.5$ ,  $a = 2$ ,  $n = 30$

cycle =  $\frac{2(1.1^{30} - 1)}{(1.1 - 1)}$

**M1** Attempt sum of GP,  $r = 1.1$ ,  $a = 2$ ,  $n = 30$

$= 329.0 \text{ km}$

total = 666 km

**A1** Obtain 666 or 667 km

**4**

**Jun 2010, Q4**

4 (i)  $u_1 = 6, u_2 = 11, u_3 = 16$

**B1** 1 State 6, 11, 16

(ii)  $S_{40} = \frac{40}{2} (2 \times 6 + 39 \times 5)$   
 $= 4140$

**M1** Show intention to sum the first 40 terms of a sequence

**M1** Attempt sum of their AP from (i), with  $n = 40$ ,  $a =$  their  $u_1$  and  $d =$  their  $u_2 - u_1$

**A1** 3 Obtain 4140

(iii)  $w_3 = 56$

**B1** State or imply  $w_3 = 56$

$5p + 1 = 56$  or  $6 + (p - 1) \times 5 = 56$

**M1** Attempt to solve  $u_p = k$

$p = 11$

**A1** 3 Obtain  $p = 11$

**7**

**Jun 2010, Q9**

(i)	$ar = a + d, ar^3 = a + 2d$ $2ar - ar^3 = a$ $ar^3 - 2ar + a = 0$ $r^3 - 2r + 1 = 0$ A.G.	M1	Attempt to link terms of AP and GP, implicitly or explicitly.
		M1	Attempt to eliminate $d$ , implicitly or explicitly, to show given equation.
		A1	3 Show $r^3 - 2r + 1 = 0$ convincingly
(ii)	$f(r) = (r-1)(r^2 + r - 1)$  $r = \frac{-1 \pm \sqrt{5}}{2}$  Hence $r = \frac{-1 + \sqrt{5}}{2}$	B1	Identify $(r-1)$ as factor or $r = 1$ as root
		M1*	Attempt to find quadratic factor
		A1	Obtain $r^2 + r - 1$
		M1d*	Attempt to solve quadratic
		A1	5 Obtain $r = \frac{-1 + \sqrt{5}}{2}$ only
(iii)	$\frac{a}{1-r} = 3 + \sqrt{5}$  $a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$  $a = 9/2 - 5/2$ $a = 2$	M1	Equate $S_\infty$ to $3 + \sqrt{5}$
		A1	Obtain $\frac{a}{1 - (\frac{-1 + \sqrt{5}}{2})} = 3 + \sqrt{5}$
		M1	Attempt to find $a$
		A1	4 Obtain $a = 2$
			<b>12</b>

**Jan 2012, Q6**

(i)	$u_1 = 80$  $u_2 = 75, u_3 = 70$	B1	State 80
		B1 [2]	State 75 and 70
(ii)	$S_{20} = \frac{20}{2}(2 \times 80 + 19 \times -5)$ $= 650$	M1	Show intention to sum 1 <sup>st</sup> 20 terms of an arithmetic sequence
		M1	Attempt use of correct sum formula for an AP, with $n = 20, a = 80, d = \pm 5$
		A1 [3]	Obtain 650

<p><b>(iii)</b></p>	$r = \frac{60}{80} = 0.75$ $u_p = 80 \times 0.75^2 = 45$ $85 - 5p = 45$ $p = 8$	<p><b>M1*</b></p> <p><b>A1</b></p> <p><b>M1d*</b></p> <p><b>A1</b></p> <p><b>[4]</b></p>	<p>Attempt to find <math>u_p</math></p> <p>Obtain 45</p> <p>Attempt to solve <math>85 - 5p = k</math></p> <p>Obtain <math>p = 8</math></p>
<p><b>(iv)</b></p>	$S_\infty = \frac{80}{1 - 0.75}$ $= 320$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>[2]</b></p>	<p>Use correct formula for sum to infinity</p> <p>Obtain 320</p>



(i)	$S_{30} = \frac{30}{2} (2 \times 6 + 29 \times 1.8)$	M1	Use $d = 1.8$ in AP formula
		A1	Correct unsimplified $S_{30}$
	$= 963$	A1 [3]	Obtain 963
(ii)	$r = \frac{7.8}{6} = 1.3$	M1	Use $r = 1.3$ in GP formula
	$\frac{6(1-1.3^N)}{1-1.3} \leq 1800$	A1	Correct unsimplified $S_N$
	$1 - 1.3^N \geq -90$	M1	Link sum of GP to 1800 and attempt to rearrange to $1.3^N \leq k$
$1.3^N \leq 91 \quad \mathbf{AG}$	A1	Obtain given inequality	