Integration Involving Trigonometric Functions MS

Q1, (OCR 4724, Jun 2006, Q8i)

Integration method			
Attempt to change $\cos^2 6x$ into $f(\cos 12x)$	**M1**		
$\cos^2 6x = \frac{1}{2}(1+\cos 12x)$	**A1**		with $\cos^2 6x$ as the subject of the formula
$\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$	**A1**		**AG** Accept $\frac{1}{2}\left(x+\frac{1}{12}\sin 12x\right)$
Differentiation method			
Differentiate RHS producing $\frac{1}{2}+\frac{1}{2}\cos 12x$ ---(E)	**B1**		
Attempt to change $\cos 12x$ into $f(\cos 6x)$	**M1**		Accept $+/-2\cos^2 6x +/-1$
Simplify (E) WWW to $\cos^2 6x$ + satis finish	**A1**	**3**	

Q32, (OCR 4724, Jun 2012, Q7)

Satisfactory start method eg <u>attempt</u> square of $(1-\sin 3x)$	M1	Not e.g. $\frac{(1-\sin 3x)^3}{3}$.
[N.B. The squaring process might include a term $\sin^2 9x$]		
The next **2** marks are awarded for integrating - $2\sin 3x$		
Obtain $\int -2\sin 3x\,dx = \frac{2}{3}\cos 3x$	*A1	
Obtain $-\frac{2}{3}$ or $(...+0...)-(...+\frac{2}{3}...)$	A1dep*	
The next **3** marks are awarded for integrating $\sin^2 3x$		or for integrating $\sin^2 ax$ where $a = 6$ or 9 **only**
Use $\sin^2 3x = k(+/-1+/-\cos 6x)$	M1	$\sin^2 ax = k(+/-1+/-\cos 2ax)$
Correct version $= \frac{1}{2}(1-\cos 6x)$	A1	Correct $= \frac{1}{2}(1-\cos 2ax)$
$\int \cos 6x\,dx = \frac{1}{6}\sin 6x$, seen anywhere, indep	B1	or $\int \cos 2ax\,dx = \frac{1}{2a}\sin 2ax$
Final answer $= \frac{1}{4}\pi + \textit{their} -\frac{2}{3}$	A1	Check that the $\frac{1}{4}\pi$ is from $\left[\frac{3}{2}x - \frac{1}{12}\sin 6x\right]_0^{\frac{1}{6}\pi}$
	[7]	

Q3, (OCR 4724, Jun 2016, Q2)

$\cos 8x$ seen in integrand	**M1**	
$F[x] = Ax + B\sin 8x$ oe	**M1***	A and B are non-zero constants
$F[x] = 6x - \frac{3}{8}\sin 8x$	**A1**	
$F[\frac{1}{8}\pi] - F[\frac{1}{16}\pi]$	**M1*dep**	
$\frac{3}{8}\pi + \frac{3}{8}$ oe	**A1**	
	[5]	

Q4, (OCR 4724, Jun 2013, Q5)

(i)	$\dfrac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$	M1	Combine (or write as 2 separate fractions) using common denominator
	$= \dfrac{2\tan x}{1-\tan^2 x} = \tan 2x$ Answer Given	A1	$\dfrac{2\tan x}{1-\tan^2 x}$ essential stage N.B. If $\tan x$ changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles
		[2]	
(ii)	$\int \tan 2x\, dx = \lambda \ln(\sec 2x)$ or $\mu \ln(\cos 2x)$ [= F(x)]	M1	
	$\lambda = \dfrac{1}{2}$ or $\mu = -\dfrac{1}{2}$	A1	
	their $F[\dfrac{\pi}{6}]$ – their $F[\dfrac{\pi}{12}]$	M1	dependent on attempt at integration……..
	$\frac{1}{2}\ln 2 - \frac{1}{2}\ln\dfrac{2}{\sqrt{3}}$ oe	A1	i.e. any correct but probably unsimplified numerical version
	$\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{4}\ln 3$ or $\ln 3^{1/4}$ or $\frac{1}{2}\ln\dfrac{6}{2\sqrt{3}}$ oe ISW	+A1	i.e. any correct version in the form $a \ln b$
		[5]	

Q5, (OCR 4724, Jan 2010, Q3)

Use $\cos 2x = a\cos^2 x + b / \pm \cos^2 x - \sin^2 x / 1 - 2\sin^2 x$	*M1	
Obtain $\lambda + \mu\sec^2 x$	dep*M1	using 'reasonable' Pythag attempt
$\int \lambda + \mu\sec^2 x\, dx = \lambda x + \mu \tan x$	A1	(λ or μ may be 0 here/prev line)
Obtain correct result $2x - \tan x$	A1	no follow-through
$\dfrac{1}{6}\pi - \sqrt{3} + 1$ ISW	A1	exact answer required
	5	

Q6, (OCR 4724, Jan 2011, Q3)

(i)	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x=\frac{1}{\cos x}$
	Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv-v\,du}{v^2}$ & wrong trig signs
	Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
	Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. Not just = sec *x*.tan *x*
(ii)	Use $\cos 2x=+/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm(\cos^2 x-\sin^2 x)$
	Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
	Evidence that $\frac{\tan x}{\cos x}=\sec x\tan x$ or $\int\frac{\tan x}{\cos x}dx=\sec x$	B1	irrespective of any const multiples
	$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for *x* except final line

Q7, (OCR 4724, Jun 2014, Q4)

$\int\frac{\cos 2x}{1+\sin 2x}(dx)$	B1* B1*	$\cos 2x = 1 - 2\sin^2 x$ or (1 +)sin2*x* = (1 +) 2sin*x*cos*x* seen numerator and denominator both correct in the integral soi	if B0B0M0A0, SC4 for F[*x*] = ½ln(1 + 2sin*x*cos*x*) or ½ln(1 + sin2*x*) final mark may still be awarded
F[*x*]= *k*ln(1 + sin2*x*) soi	M1dep*	or *k*ln(1 + *u*) or *k*ln(*u*) following their substitution www	
k = ½	A1	correct *k* for their substitution	
$\frac{1}{2}\ln(1+\sin(\frac{\pi}{2}))-\frac{1}{2}\ln(1+0)$ = ½ln2	A1 AG	correct use of limits www	minimum working: ½ln2 − ½ln1 or ½ln(1 + 1) oe
	[5]		

Q8, (OCR 4724, Jun 2015, Q6)

(i)		$\frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$ may be implied by $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ eg $= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ and completion to $\frac{-1}{\sin^2 x}$ **AG**	**M1** **A1** **[2]**	or $-\sin x \times \frac{1}{\sin x} + \cos x \times -(\sin x)^{-2} \times \cos x$ oe eg $= \frac{-\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$ oe and completion to $\frac{-1}{\sin^2 x}$	allow sign errors only if **M0**, **SC1** for just $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$ need to see at least two correct, constructive steps and statement of given answer for **A1** NB $\sin^2 x + \cos^2 x = 1$ seen may be a constructive intermediate step

(ii)		$\cos 2x = 2\cos^2 x - 1$ substituted in numerator	**M1**	or alternative form of double angle formula plus Pythagoras leading to no term in $\sin^2 x$ in numerator	may be awarded if not seen as part of fraction
		$\sin 2x = 2\sin x\cos x$ substituted in denominator	**M1**		
		$\dfrac{\sqrt{2}\cos x}{2\sin^2 x\cos x}$	**A1**		NB $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{1}{\sqrt{2}\sin^2 x}\,dx$
		$F[x] = \pm k\dfrac{\cos x}{\sin x}$	**M1***	k must not be obtained from square rooting a negative number	NB $-\dfrac{\cos x}{\sqrt{2}\sin x}$
		$F\left[\frac{1}{4}\pi\right] - F\left[\frac{1}{6}\pi\right]$	**M1dep***	eg $\dfrac{-\cos \pi/4}{\sqrt{2}\times\sin \pi/4} - \dfrac{-\cos \pi/6}{\sqrt{2}\times\sin \pi/6}$	eg $\dfrac{-1/\sqrt{2}}{\sqrt{2}\times 1/\sqrt{2}} - \dfrac{-\sqrt{3}/2}{\sqrt{2}\times 1/2}$
		$= \frac{1}{2}(\sqrt{6} - \sqrt{2})$ www **AG**	**A1**		at least one correct intermediate step following substitution needed as well as statement of given result eg $-\dfrac{\sqrt{2}}{2}(1-\sqrt{3})$
			[6]		

Q9, (OCR 4724, Jan 2008, Q7)

(i) Perform an operation to produce an equation connecting A and B (or possibly in A or in B)	M1		Probably substituting value of θ, or comparing coefficients of $\sin x$, and/or $\cos x$
$A = 2$	A1		
$B = -2$	A1	**3**	WW scores 3
(ii) Write $4\sin\theta$ as $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)$ and re-write integrand as $A+\dfrac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}$	M1		A and B need not be numerical – but, if they are, they should be the values found in (i).
$\int A\,d\theta = A\theta$	√B1		general or numerical
$\int \dfrac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}\,d\theta = B\ln(\sin\theta+\cos\theta)$	√A2		general or numerical
Produce $\frac{1}{4}A\pi + B\ln\sqrt{2}$ f.t. with their A,B	√A1	**5**	Expect $\frac{1}{2}\pi - \ln 2$ (Numerical answer only)