Integration By Substitution Exam Questions (From OCR 4724)

Q1, (Jun 2005, Q4)

(i) Show that the substitution $x = \tan\theta$ transforms $\int \frac{1}{(1+x^2)^2}\,dx$ to $\int \cos^2\theta\,d\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1+x^2)^2}\,dx$. [4]

Q2, (Jan 2006, Q6)

(i) Show that the substitution $x = \sin^2\theta$ transforms $\int \sqrt{\frac{x}{1-x}}\,dx$ to $\int 2\sin^2\theta\,d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}}\,dx$. [5]

Q3, (Jan 2008, Q10i)

(i) Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx.$$ [6]

Q4, (Jun 2008, Q8)

(i) Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}}\,dx$ to $\int \frac{2t}{(t+1)^2}\,dt$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x+\sqrt{2x-1}}\,dx$. [4]

Q5, (Jan 2009, Q5)

(i) Show that the substitution $u = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})}\,dx$ to $\int \frac{2}{u(1+u)}\,du$. **[3]**

(ii) Hence find the exact value of $\int_1^9 \frac{1}{x(1+\sqrt{x})}\,dx$. **[5]**

Q6, (Jan 2010, Q4)

Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^{e} \frac{1}{t(2+\ln t)^2}\,dt.$$ **[6]**

Q7, (Jan 2011, Q5)

In this question, I denotes the definite integral $\int_2^5 \frac{5-x}{2+\sqrt{x-1}}\,dx$. The value of I is to be found using two different methods.

(i) Show that the substitution $u = \sqrt{x-1}$ transforms I to $\int_1^2 (4u - 2u^2)\,du$ and hence find the exact value of I. **[5]**

(ii) (a) Simplify $(2+\sqrt{x-1})(2-\sqrt{x-1})$. **[1]**

(b) By first multiplying the numerator and denominator of $\frac{5-x}{2+\sqrt{x-1}}$ by $2-\sqrt{x-1}$, find the exact value of I. **[3]**

Q8, (Jan 2013, Q6)

Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5}\,dx$. **[7]**

Q9, (Jun 2016, Q6)

Use the substitution $u = x^2 - 2$ to find $\int \frac{6x^3+4x}{\sqrt{x^2-2}}\,dx$. **[6]**

Q10, (Jun 2017, Q9)

Use the substitution $u = 1 + \ln x + x$ to find $\int \frac{3(x+1)(1-\ln x - x)}{x(1+\ln x + x)}\,dx$. **[6]**
