Integration By Parts MS (From OCR 4724)

Q1, (Jun 2005, Q2)

	M1	For attempt at parts going correct way (u = x, dv = cos x and f(x) +/–∫g(x) (dx)
$x \sin x - \int \sin x \, dx$	A1	For both terms correct
$(= x \sin x + \cos x)$	B1	Indic anywhere that $\int \sin x \, dx = -\cos x$
	M1	For correct method of limits
Answer = $\frac{1}{2}\pi - 1$	A1 **5**	For correct exact answer ISW **5**

Q2, (Jan 2006, Q4)

(i) Parts using correct split of $u = x, \frac{dv}{dx} = \sec^2 x$	M1		1st stage result of form $f(x) +/- \int g(x) dx$
$x \tan x - \int \tan x \, dx$	A1		Correct 1st stage
$\int \tan x \, dx = -\ln \cos x$ or $\ln \sec x$	B1		
$x \tan x + \ln \cos x + c$ or $x \tan x - \ln \sec x + c$	A1	**4**	
(ii) $\tan^2 x = +/- \sec^2 x +/- 1$	M1		or $\sec^2 x = +/- 1 +/- \tan^2 x$
$\int x \sec^2 x \, dx - \int x \, dx$ s.o.i.	A1		Correct 1st stage
$x \tan x + \ln \cos x - \frac{1}{2}x^2 + c$	A1√	**3**	f.t. their answer to part (i) $- \frac{1}{2}x^2$

Q3, (Jan 2007, Q2)

Use parts with $u = \ln x, dv = x$	M1		& give 1st stage in form $f(x) +/- \int g(x)(dx)$
Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2}x^2 (dx)$	A1		or $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x (dx)$
$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$ (+c)	A1		
Use limits correctly	M1		
Exact answer $2 \ln 2 - \frac{3}{4}$	A1	**5**	AEF ISW

Q4, (Jun 2007, Q2)

Use parts with $u = x^2, dv = e^x$	*M1		obtaining a result $f(x) +/- \int g(x)(dx)$
Obtain $x^2 e^x - \int 2xe^x (dx)$	A1		
Attempt parts again with $u = (-)(2)x, dv = e^x$	M1		
Final = $(x^2 - 2x + 2)e^x$ AEF incl brackets	A1		s.o.i. eg $e + (-2x + 2)e^x$
Use limits correctly throughout	dep*M1		Tolerate (their value for $x = 1$) (-0)
$e^{(1)} - 2$ ISW Exact answer only	A1	**6**	Allow 0.718 → M1

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Q5, (Jan 2010, Q8)

(i)	$-\sin x e^{\cos x}$	B1 **1**	
(ii)	$\int \sin x e^{\cos x} dx = -e^{\cos x}$	B1	anywhere in part (ii)
	Parts with split $u = \cos x, dv = \sin x e^{\cos x}$	M1	result $f(x) +/- \int g(x) dx$
	Indef Integ, 1st stage $-\cos x e^{\cos x} - \int \sin x e^{\cos x} dx$	A1	accept … $-\int -e^{\cos x}.-\sin x dx$
	Second stage = $-\cos x e^{\cos x} + e^{\cos x}$	*A1	
	Final answer = 1	dep*A2 **6**	
		7	

Q6, (Jun 2010, Q9i)

Attempt to multiply out $(x + \cos 2x)^2$	M1	Min of 2 correct terms
<u>Finding $\int 2x \cos 2x\, dx$</u>		
Use $u = 2x, dv = \cos 2x$	M1	1st stage $f(x) +/- \int g(x) dx$
1st stage $x \sin 2x - \int \sin 2x\, dx$	A1	
$\therefore \int 2x \cos 2x\, dx = x \sin 2x + \frac{1}{2} \cos 2x$	A1	
<u>Finding $\int \cos^2 2x\, dx$</u>		
Change to $k \int +/-1 +/- \cos 4x\, dx$	M1	where $k = \frac{1}{2}$, 2 or 1
Correct version $\frac{1}{2} \int 1 + \cos 4x\, dx$	A1	
$\int \cos 4x\, dx = \frac{1}{4} \sin 4x$	B1	seen anywhere in this part
Result = $\frac{1}{2} x + \frac{1}{8} \sin 4x$	A1	
(i) ans $= \frac{1}{3} x^3 + x \sin 2x + \frac{1}{2} \cos 2x + \frac{1}{2} x + \frac{1}{8} \sin 4x\ (+ c)$	A1 **9**	Fully correct

Q7, (Jun 2013, Q2)

$u = \ln 3x$ and dv or $\frac{dv}{dx} = x^8$	M1	integ by parts as far as $f(x) +/- \int g(x)(dx)$	If difficult to assess, x^8 must be integrated, so look for term in x^9
$\frac{d}{dx}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$	B1	stated or clearly used	
$\frac{x^9}{9}\ln 3x - \int \frac{x^9}{9}$ their $\frac{du}{dx}(dx)$ FT	√A1	i.e. correct understanding of 'by parts'…	..even if ln(3x) incorrectly differentiated
Indication that $\int kx^8 dx$ is required	M1	i.e. before integrating, product of terms must be taken	The product may already have been indicated on the previous line
$\frac{x^9}{9}\ln 3x - \frac{x^9}{81}$ or $\frac{1}{9}x^9\left(\ln 3x - \frac{1}{9}\right)$ ISW (+c) <u>cao</u>	A1	$\frac{1}{9}\frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis	
	[5]		
<u>If candidate manipulates ln(3x) first of all</u>			
ln(3x) = ln 3 + ln x	B1		If, however, ln(3x) is said to be ln 3.ln x, then B0 followed by possible M1 A1 A1 in line with alternative solution on LHS, where the 'M' mark is for dealing with $\int x^8 \ln x\, dx$ 'by parts' in the right order and the 2 @ A1 are for correct results.
$u = \ln x$ and $dv = x^8$	M1	In order to find $\int x^8 \ln x\, dx$:	
$\frac{x^9}{9}\ln x - \int \frac{x^9}{9}.\frac{1}{x}(dx)$ or better	A1		
$\frac{x^9}{9}\ln x - \frac{x^9}{81}$	A1		
Their $\int x^8 \ln x\, dx + \frac{x^9}{9}\ln 3$ (+ c) FT ISW	√A1		

Q8, (Jun 2014, Q8)

(i)		t^2 in quotient and $t^3 + 2t^2$ seen $-2t$ in quotient and $-2t^2 - (-2t^2 - 4t) = 4t$ seen completion to obtain correct quotient and remainder identified www	B1 B1 B1 **[3]**	or $\frac{t(t^2-4)+4t}{(t+2)}$ $\frac{t(t+2)(t-2)}{(t+2)} + \frac{4t}{t+2}$ $t(t-2) + \frac{4(t+2)-8}{t+2}$	or $\frac{(t+2)^3 - 6t^2 - 12t - 8}{(t+2)}$ $\frac{(t+2)^3}{(t+2)} - \frac{6((t+2)^2 - 4t - 4) + 12t + 8}{(t+2)}$ oe $(t+2)^2 - 6(t+2) + \frac{12t+16}{t+2}$ oe $= t^2 + 4t + 4 - 6t - 12 + \frac{12(t+2)-8}{t+2}$ oe both steps needed for final B1
(i)		alternatively $\frac{t^3}{t+2} \equiv At^2 + Bt + C + \frac{D}{(t+2)}$ equate coefficients to obtain correctly $A = 1, 0 = 2A + B$ and $B = -2$ www $0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www	B1 B1 B1 **[3]**	or $t^3 \equiv (At^2 + Bt + C)(t+2) + D$	or B1 for $\frac{t^2(t+2) - 2t^2}{(t+2)}$ B1 for $t^2 + \frac{-2t(t+2)+4t}{(t+2)}$ B1 for $t^2 - 2t + \frac{4(t+2)-8}{(t+2)}$

(ii)	integration by parts with $u = \ln(t+2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt)$	M1*	f(t) must include t^3 and g(t) must **not** include a logarithm	ignore spurious dx etc
	$2t^3\ln(t+2) - \int \frac{2t^3}{t+2}(dt)$ cao	A1		alternatively, following $u = t+2$
	result from part (i) seen in integrand; must follow award of at least first M1	M1*	no integration required for this mark	$\int 2(u^2 - 6u + 12 - \frac{8}{u})du$ oe
	$F[t] = 2t^3\ln(t+2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16\ln(t+2)$	A1	$2t^3\ln(t+2) - \frac{2t^3}{3} + 2t^2 - 8t + 16\ln(t+2)$	$\frac{2u^3}{3} - 6u^2 + 24u - 16\ln u$ and $2t^3\ln(t+2)$
	their F[2] – F[1]	M1dep*	at least one of their terms correctly integrated	NB limits following substitution are $u = 4$ and $u = 3$
	–6⅔ – 18ln3 + 32ln4 oe cao	A1		
		[6]		

Q9, (Jun 2016, Q4)

$Ax^{\frac{2}{3}}\ln x - \int Bx^{\frac{2}{3}} \times \frac{1}{x}dx$ oe	**M1***	A and B are non-zero constants;	
$\frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{\frac{2}{3}} \times \frac{1}{x}dx$	**A1**	ignore + c	NB $\frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{-\frac{1}{3}}dx$ Allow both marks if dx omitted
$F[x] = \frac{3}{2}x^{\frac{2}{3}}\ln x - \frac{3/2}{2/3}x^{\frac{2}{3}}$	**A1**	ignore limits for first three marks	
F[8] – F[1]	**M1*dep**	and also dependent on integration of their $\frac{3}{2}x^{-\frac{1}{3}}$	
$18\ln 2 - \frac{27}{4}$ cao	**A1**		**NB A0** for $6\ln 8 - \frac{27}{4}$
	[5]		