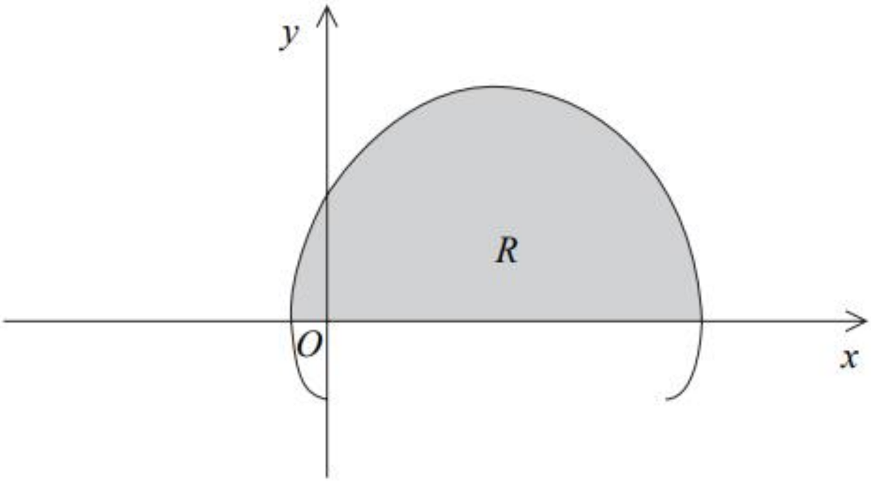
Integrating Parametric Functions (From Edexcel 6666)

Q1, (Jan 2006, Q8)

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \quad y = 1 - 2\cos t, \quad 0 \leqslant t \leqslant 2\pi.$$

(a) Show that the curve crosses the x-axis where $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

(b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)

Q2, (Jan 2008, Q7)

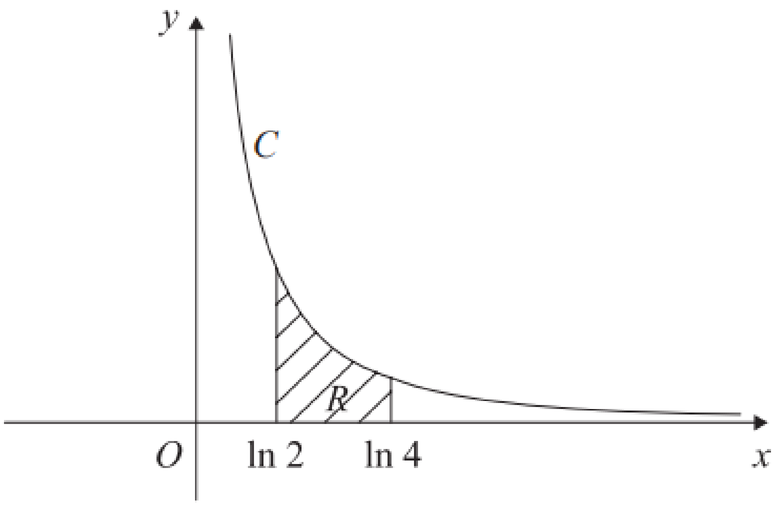


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

Q3, (Jun 2008, Q8)

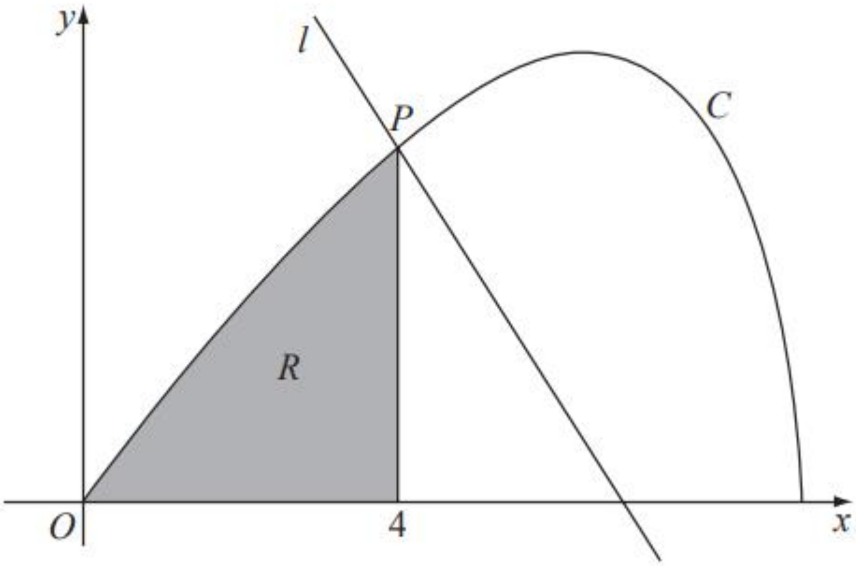


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

Q4, (Jan 2010, Q7)

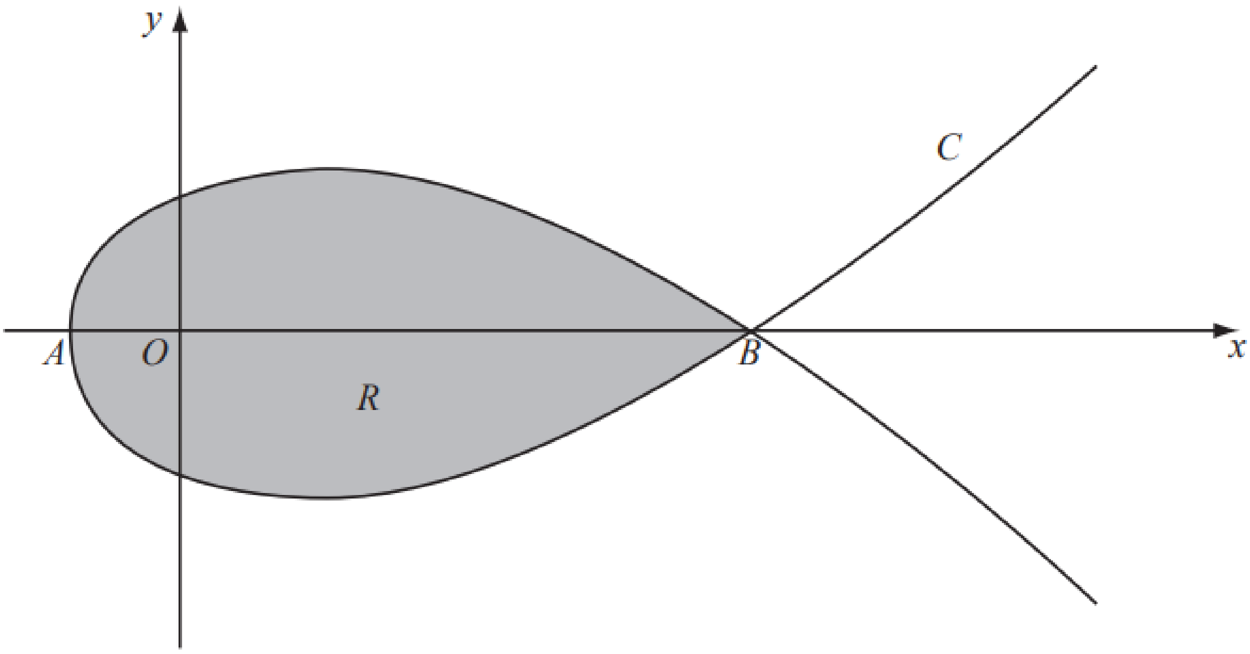


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \qquad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

Q5, (Jan 2013, Q5)

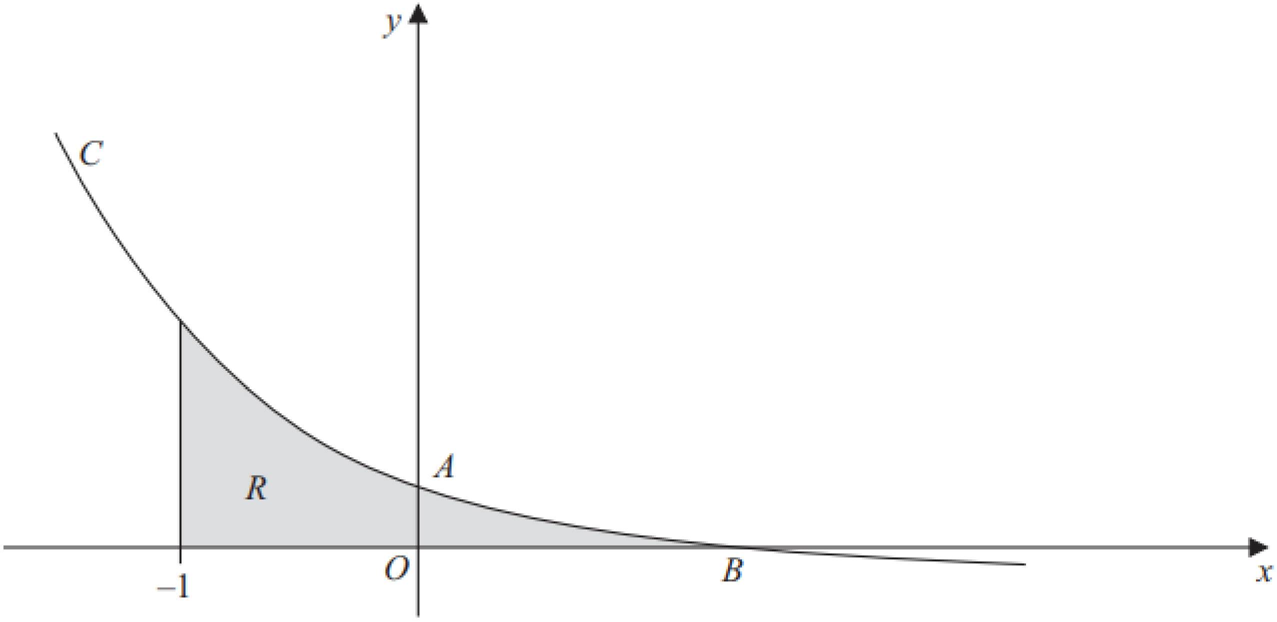


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates $(0, 3)$.

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

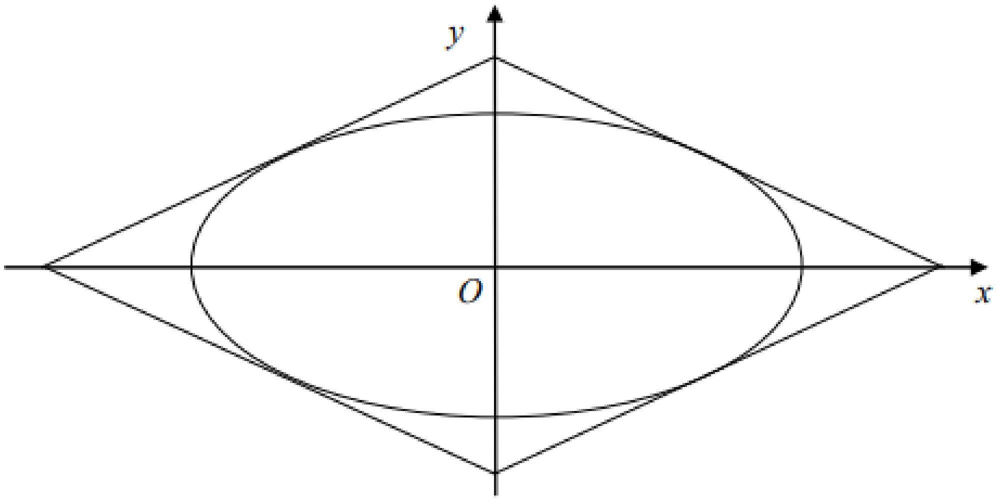
The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

Q6, (Edexcel 6666, Sample Paper A3, Q8)

Figure 1



A table top, in the shape of a parallelogram, is made from two types of wood. The design is shown in Fig. 1. The area inside the ellipse is made from one type of wood, and the surrounding area is made from a second type of wood.

The ellipse has parametric equations,

$$x = 5\cos\theta, \quad y = 4\sin\theta, \quad 0 \le \theta < 2\pi.$$

The parallelogram consists of four line segments, which are tangents to the ellipse at the points where $\theta = \alpha$, $\theta = -\alpha$, $\theta = \pi - \alpha$, $\theta = -\pi + \alpha$.

(*a*) Find an equation of the tangent to the ellipse at $(5\cos\alpha, 4\sin\alpha)$, and show that it can be written in the form

$$5y\sin\alpha + 4x\cos\alpha = 20.$$

(4)

(*b*) Find by integration the area enclosed by the ellipse.

(4)

(*c*) Hence show that the area enclosed between the ellipse and the parallelogram is

$$\frac{80}{\sin 2\alpha} - 20\pi.$$

(4)

(*d*) Given that $0 < \alpha < \frac{\pi}{4}$, find the value of α for which the areas of two types of wood are equal.

(3)

Q7, (Edexcel 6666, Sample Paper A6, Q4)

Figure 1

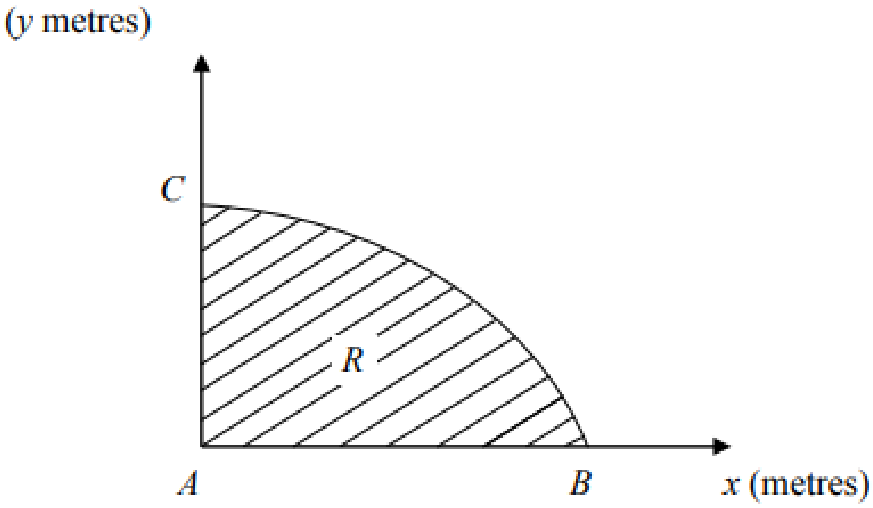


Figure 1 shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates $(0, 0)$, $(3\pi^2, 0)$ and $(0, 30)$ respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(a) Find an estimate for the area of the cross-section R using this approximation.

(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30 \sin 2t, \quad \frac{\pi}{4} \leq t \leq \frac{\pi}{2}.$$

(b) Find the exact area of the cross-section R.

(7)

(c) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (a).

(2)