Implicit Differentiation Exam Questions (From OCR 4724)

Q1, (Jun 2007, Q6)

(i) $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
Using $d(uv) = u\,dv + v\,du$ for the $(3)xy$ term	M1		
$\frac{d}{dx}(x^2 + 3xy + 4y^2) = 2x + 3x\frac{dy}{dx} + 3y + 8y\frac{dy}{dx}$	A1		
Solve for $\frac{dy}{dx}$ & subst $(x, y) = (2,3)$	M1		or v.v. Subst now or at normal eqn stage; (M1 dep on either/both B1 M1 earned)
$\frac{dy}{dx} = -\frac{13}{30}$	A1		Implied if grad normal = $\frac{30}{13}$
Grad normal = $\frac{30}{13}$ follow-through	√B1		This f.t. mark awarded only if numerical
Find equ <u>any</u> line thro (2,3) with <u>any</u> num grad	M1		
$30x - 13y - 21 = 0$ AEF	A1	8	No fractions in final answer **8**

Q2, (Jan 2008, Q6)

$\frac{d}{dx}(x^2y) = x^2\frac{dy}{dx} + 2xy$	B1		s.o.i.;
$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$	B1		
Substitute $(x,y) = (1,1)$ and solve for $\frac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
	M1		dep on either/both B1 earned]
$\frac{dy}{dx} = -\frac{11}{7}$ WWW	A1		Implied if grad normal = $\frac{7}{11}$
Gradient normal = $-\frac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
$7x - 11y + 4 = 0$ AEF	A1	6	No fractions in final answer.

Q3, (Jan 2009, Q8)

(i)	$\frac{d}{dx}\left(y^3\right) = 3y^2 \frac{dy}{dx}$	B1	
	Consider $\frac{d}{dx}(xy)$ as a product	M1	
	$= \quad x\frac{dy}{dx} + y$	A1	Tolerate omission of '6'
	$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ ISW AEF	A1 (4)	
(ii)	$x^3 = 2^4$ or 16 and $y^3 = 2^5$ or 32	*B1	
	Satisfactory conclusion	dep* B1	**AG**
	Substitute $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$	M1	or the numerator of $\frac{dy}{dx}$
	Show or use calc to demo that num = 0, ignore denom **AG**	A1 (4)	
(iii)	Substitute (a, a) into eqn of curve	M1	& attempt to state '$a = \ldots$'
	$a = 3$ only with clear ref to $a \neq 0$	A1	
	Substitute $(3,3)$ or (their a, their a) into their $\frac{dy}{dx}$	M1	
	-1 only WWW	A1 (4)	from (their a,their a)
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Q4, (Jun 2009, Q8)

(i)	$\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$	B1	
	$\frac{d}{dx}(uv) = u\ dv + v\ du$ used on $(-7)xy$	M1	
	$\frac{d}{dx}\left(14x^2 - 7xy + y^2\right) = 28x - 7x\frac{dy}{dx} - 7y + 2y\frac{dy}{dx}$	A1	$(=0)$
	$2y\frac{dy}{dx} - 7x\frac{dy}{dx} = 7y - 28x \rightarrow \frac{dy}{dx} = \frac{28x-7y}{7x-2y}$ www AG	A1 **4**	As AG, intermed step nec

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(ii)	Subst $x = 1$ into eqn curve & solve quadratic eqn in y	M1	$(y = 3$ or $4)$
	Subst $x = 1$ and (one of) their y-value(s) into given $\frac{dy}{dx}$	M1	$\left(\frac{dy}{dx} = 7 \text{ or } 0\right)$
	Find eqn of tgt, with their $\frac{dy}{dx}$, going through (1, their y)	*M1	using (one of) y value(s)
	Produce either $y = 7x - 4$ or $y = 4$	A1	
	Solve simultaneously their two equations	dep*M1	provided they have two
	Produce $x = \frac{8}{7}$	A1 **6**	

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Q5, (Jun 2010, Q5)

$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1	Implied by e.g., $4x\frac{dy}{dx} + y$
$\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$	B1	
Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$	M1	
Produce only $2x + 4y = 0$ (though AEF acceptable)	*A1	without any error seen
Eliminate x or y from curve eqn & eqn(s) just produced	M1	
Produce either $x^2 = 36$ or $y^2 = 9$	dep* A1	Disregard other solutions
$(\pm 6, \mp 3)$ AEF, as the only answer ISW	dep* A1	Sign aspect must be clear

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Q6, (Jan 2013, Q3)

For attempt at product rule on xy^2	M1	or changing equation to $y^2 = x + x^{-1}$
$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	soi in the differentiating process
$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$ or $\frac{1 - x^{-2}}{2y}$	A1	Award B1 for $(\pm)\frac{1}{2}(x + x^{-1})^{-\frac{1}{2}}(1 - x^{-2})$
Stationary point $\rightarrow$ (their) $\frac{dy}{dx} = 0$ soi	M1	
$x^2 = 1$ <u>or</u> $y^2 = 2$ <u>or</u> $y^4 = 4$	A1	Ignore any other values
$(1, \sqrt{2})$, $(1, -\sqrt{2})$	A1,A1	Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$
	[7]	

Q7, (Jun 2015, Q7)

LHS is $k(x+y)(1+\frac{dy}{dx})$	**M1**	or $2x+2y\frac{dy}{dx}+ky+kx\frac{dy}{dx}$ k is any positive integer	some terms may appear on RHS with signs reversed
$k = 2$	**A1**		if **M0** in middle scheme, **SC1** for **three terms out of four** completely correct with $k = 2$
$2y\frac{dy}{dx}$ on RHS from differentiating y^2	**B1**		may appear on LHS with sign reversed
$y^2 + Kxy\frac{dy}{dx}$ on RHS	**M1**	K is any positive integer	NB $K = 2$; may appear on LHS with signs reversed
obtains a value of y from eg $(1+y)^2 = 1\times y^2$ oe	**M1**	allow even if follows incorrect manipulation	NB $y = -0.5$
substitution of $x = 1$ and their y dependent on at least two correct terms seen following differentiation, even if follows subsequent incorrect manipulation	**M1**	may be implied by $1+\frac{dy}{dx}=\frac{1}{4}-\frac{dy}{dx}$	or $\frac{dy}{dx}=\frac{2-1-0.25}{-1-2+1}$ NB $\frac{dy}{dx}=\frac{2x+2y-y^2}{2xy-2x-2y}$
$\frac{dy}{dx}=-\frac{3}{8}$ oe cao	**A1** **[7]**		-0.375

Q8, (Jun 2016, Q3)

$2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	from differentiation of y^2	
$\sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x$	**M1**	correct use of Product Rule	allow sign error or one incorrect coefficient
$\sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x-\dfrac{1}{x^2}+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	**A1**		
$(\sin 2x+2y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x^2}-2y\cos 2x$ oe	**M1**	collection of like terms on separate sides, need not be factorised	must be two terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
$[\dfrac{\mathrm{d}y}{\mathrm{d}x}=]\dfrac{1-2x^2y\cos 2x}{(\sin 2x+2y)x^2}$ oe isw	**A1**	eg $[\dfrac{\mathrm{d}y}{\mathrm{d}x}=]\dfrac{x^{-2}-2y\cos 2x}{(\sin 2x+2y)}$	**A0** for eg $y=\ldots$
	[5]		

Q9, (Jun 2014, Q6)

$3y^2\frac{dy}{dx}$	B1	or $2x\frac{dx}{dy}$	if B0B0 M0
$2x-12\frac{dy}{dx}-8$	B1	$3y^2-8\frac{dx}{dy}-12$	SC2 for $\frac{dy}{dx}=$
their $3y^2\frac{dy}{dx}-12\frac{dy}{dx}=8-2x$ soi must be two terms on each side and must follow from RHS $=0$	M1	their $2x\frac{dx}{dy}-8\frac{dx}{dy}=-3y^2+12$ must be two terms on each side must follow from RHS $=0$	$\frac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x+8+12\frac{dy}{dx})$ M1 may be earned for setting correct denominator equal to 0
$\frac{dy}{dx}=\frac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\frac{dx}{dy}=0$ is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$	
their $3y^2-12=0$	M1*		$x\neq 4$ not required
$y=(\pm)\,2$	A1	A0 if $\frac{dy}{dx}$ incorrect	
substitution of their positive y value in original equation	M1dep*		ignore substitution of -2
$x=10, x=-2$ and no others cao	A1	A0 if $\frac{dy}{dx}$ incorrect	condone omission of formal statement of coordinates (10, 2) and (–2, 2)
	[8]		