Further Trig and Reciprocal Trig Functions (From OCR 4723)

Q1, (Jun 2005, Q7)

(i) Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]

(ii) Prove the identity $\dfrac{4\cos 2x}{1+\cos 2x} \equiv 4 - 2\sec^2 x$. [3]

(iii) Solve, for $0 < x < 2\pi$, the equation $\dfrac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$. [5]

Q2, (Jun 2006, Q5)

(i) Write down the identity expressing $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. [1]

(ii) Given that $\sin\alpha = \frac{1}{4}$ and α is acute, show that $\sin 2\alpha = \frac{1}{8}\sqrt{15}$. [3]

(iii) Solve, for $0^\circ < \beta < 90^\circ$, the equation $5\sin 2\beta \sec\beta = 3$. [3]

Q3, (Jan 2007, Q2)

It is given that θ is the acute angle such that $\sin\theta = \frac{12}{13}$. Find the exact value of

(i) $\cot\theta$, [2]

(ii) $\cos 2\theta$. [3]

Q4, (Jun 2008, Q5)

(a) Express $\tan 2\alpha$ in terms of $\tan\alpha$ and hence solve, for $0^\circ < \alpha < 180^\circ$, the equation

$$\tan 2\alpha \tan\alpha = 8.$$ [6]

(b) Given that β is the acute angle such that $\sin\beta = \frac{6}{7}$, find the exact value of

(i) $\operatorname{cosec}\beta$, [1]

(ii) $\cot^2\beta$. [2]

Q5, (Jan 2009, Q3)

(i) Express $2\tan^2\theta - \dfrac{1}{\cos\theta}$ in terms of $\sec\theta$. **[3]**

(ii) Hence solve, for $0^\circ < \theta < 360^\circ$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4.$$ **[4]**

Q6, (Jan 2010, Q2)

The angle θ is such that $0^\circ < \theta < 90^\circ$.

(i) Given that θ satisfies the equation $6\sin 2\theta = 5\cos\theta$, find the exact value of $\sin\theta$. **[3]**

(ii) Given instead that θ satisfies the equation $8\cos\theta\operatorname{cosec}^2\theta = 3$, find the exact value of $\cos\theta$. **[5]**

Q7, (Jun 2010, Q3)

(i) Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. **[3]**

(ii) Hence solve, for $-180° < \theta < 180°$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$. **[3]**

Q8, (Jan 2012, Q4)

The acute angles α and β are such that

$$2\cot\alpha = 1 \quad \text{and} \quad 24 + \sec^2\beta = 10\tan\beta.$$

(i) State the value of $\tan\alpha$ and determine the value of $\tan\beta$. **[4]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

Q9, (Jan 2013, Q2)

The acute angle A is such that $\tan A = 2$.

(i) Find the exact value of $\operatorname{cosec} A$. **[2]**

(ii) The angle B is such that $\tan(A + B) = 3$. Using an appropriate identity, find the exact value of $\tan B$. **[3]**

Q10, (Jun 2013, Q2)

Using an appropriate identity in each case, find the possible values of

(i) $\sin\alpha$ given that $4\cos 2\alpha = \sin^2\alpha$, **[3]**

(ii) $\sec\beta$ given that $2\tan^2\beta = 3 + 9\sec\beta$. **[4]**

Q11, (Jun 2014, Q2)

By first using appropriate identities, solve the equation

$$5\cos 2\theta \operatorname{cosec}\theta = 2$$

for $0° < \theta < 180°$. **[6]**

Q12, (Jun 2016, Q4)

It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27\cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of $\tan(A - B)$. **[8]**

Q13, (Jun 2016, Q9)

(i) Show that $\sin 2\theta(\tan\theta + \cot\theta) \equiv 2$. **[4]**

(ii) Hence

(a) find the exact value of $\tan\frac{1}{12}\pi + \tan\frac{1}{8}\pi + \cot\frac{1}{12}\pi + \cot\frac{1}{8}\pi$, **[3]**

(b) solve the equation $\sin 4\theta(\tan\theta + \cot\theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$, **[3]**

(c) express $(1 - \cos 2\theta)^2\left(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta\right)^3$ in terms of $\sin\theta$. **[2]**

Q14, (Jun 2017, Q4)

The angle θ, where $90° < \theta < 180°$, satisfies the equation

$$3\sec^2\theta + 10\tan\theta = 11.$$

(i) Find the value of $\tan\theta$. **[3]**

(ii) Without using a calculator, determine the value of

(a) $\tan 2\theta$, **[2]**

(b) $\cot(2\theta + 135°)$. **[3]**
