

Differentiation (Chain, Product and Quotient Rules) Introductory Exam Questions (From OCR 4723)

Q1, (Jan 2006, Q3)

(a) Attempt use of product rule	M1	[involving ... + ...]
Obtain $2x(x+1)^6 \dots$	A1	
Obtain $\dots + 6x^2(x+1)^5$	A1 3	[or equivs; ignore subsequent attempt at simplification]
 (b) Attempt use of quotient rule	M1	[or, with adjustment, product rule; allow u/v confusion]
Obtain $\frac{(x^2 - 3)2x - (x^2 + 3)2x}{(x^2 - 3)^2}$	A1	[or equiv]
Obtain -3	A1 3	[from correct derivative only]

Q2, (Jun 2006, Q1)

Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$	M1	any non-zero constant k
Obtain $2(4x+1)^{-\frac{1}{2}}$	A1	or equiv, perhaps unsimplified
Obtain $\frac{2}{3}$ for value of first derivative	A1	or unsimplified equiv
Attempt equation of tangent through (2, 3)	M1	using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
Obtain $y = \frac{2}{3}x + \frac{5}{3}$ or $2x - 3y + 5 = 0$	A1 5	or equiv involving 3 terms

Q3, (Jan 2007, Q1)

Attempt use of quotient rule to find derivative	M1	allow for numerator 'wrong way round'; or attempt use of product rule
Obtain $\frac{2(3x-1) - 3(2x+1)}{(3x-1)^2}$	A1	or equiv
Obtain $-\frac{5}{4}$ for gradient	A1	or equiv
Attempt eqn of straight line with numerical gradient	M1	obtained from their $\frac{dy}{dx}$; tangent not normal
Obtain $5x + 4y - 11 = 0$	A1 5	or similar equiv

Q4, (Jun 2007, Q1)

(i) Attempt use of product rule	M1	
Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$	A1 2	or equiv
[Or: (following complete expansion and differentiation term by term)		
Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$	B2	allow B1 if one term incorrect]
(ii) Obtain derivative of form $kx^3(3x^4 + 1)^n$	M1	any constants k and n
Obtain derivative of form $kx^3(3x^4 + 1)^{-\frac{1}{2}}$	M1	
Obtain correct $6x^3(3x^4 + 1)^{-\frac{1}{2}}$	A1 3	or (unsimplified) equiv

Q5, (Jun 2008, Q3)

Attempt use of product rule

Obtain $2x \ln x + x^2 \cdot \frac{1}{x}$

Substitute e to obtain 3e for gradient

Attempt eqn of straight line with numerical gradient

Obtain $y - e^2 = 3e(x - e)$

Obtain $y = 3ex - 2e^2$

M1 ... + ... form

A1 or equiv

A1 or exact (unsimplified) equiv

M1 allowing approx values

A1✓ or equiv; following their gradient provided obtained by diffn attempt; allow approx values

A1 in terms of e now and in requested form

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Q6, (Jan 2010, Q5)

(i) Obtain derivative of form $kx(x^2 + 1)^7$

Obtain $16x(x^2 + 1)^7$

Equate first derivative to 0 and confirm $x = 0$ or substitute $x = 0$ and verify first derivative zero

Refer, in some way, to $x^2 + 1 = 0$ having no root

M1 any constant k

A1 or equiv

M1 AG; allow for deriv of form $kx(x^2 + 1)^7$

A1 4 or equiv

(ii) Attempt use of product rule

Obtain $16(x^2 + 1)^7 + \dots$

Obtain $\dots + 224x^2(x^2 + 1)^6$

Substitute 0 in attempt at second derivative

Obtain 16

***M1** obtaining ... + ... form

A1✓ follow their $kx(x^2 + 1)^7$

A1✓ follow their $kx(x^2 + 1)^7$; or unsimplified equiv

M1 dep *M

A1 5 from second derivative which is correct at some point

9

Q7, (Jun 2010, Q1)

(i) Attempt use of product rule

Obtain $3x^2e^{2x} + 2x^3e^{2x}$

M1 producing ... + ... form

A1 2 or equiv

(ii) Attempt use of chain rule to produce $\frac{kx}{3 + 2x^2}$ form

Obtain $\frac{4x}{3 + 2x^2}$

M1 any constant k

A1 2

(iii) Attempt use of quotient rule

Obtain $\frac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$

M1 or equiv; condone u/v confusions

A1 2 or (unsimplified) equiv

[If ... + c included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of ... + c.]

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Q8, (Jan 2013, Q1)

(i)	<p><u>Either</u> Attempt use of quotient rule</p> <p>Obtain $\frac{3(2x+1)-6x}{(2x+1)^2}$ or equiv</p> <p>Substitute 2 to obtain $\frac{3}{25}$ or 0.12</p>	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x ; for M1 condone minor errors such as absence of square in denominator, absence of brackets, ...
	<p>Or Attempt use of product rule for $3x(2x+1)^{-1}$</p> <p>Obtain $3(2x+1)^{-1} - 6x(2x+1)^{-2}$ or equiv</p> <p>Substitute 2 to obtain $\frac{3}{25}$ or 0.12</p>	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'
		A1	or simplified equiv but A0 for final $\frac{3}{25}$
		[3]	
		M1	allow sign error; condone no use of chain rule
		A1	
		A1	or simplified equiv
(ii)	<p>Differentiate to obtain form $kx(4x^2+9)^n$</p> <p>Obtain $4x(4x^2+9)^{-\frac{1}{2}}$</p> <p>Substitute 2 to obtain $\frac{8}{5}$ or 1.6</p>	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)
		A1	or (unsimplified) equiv
		A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
		[3]	

Q9, (Jan 2013, Q4)

(i)	<p>Attempt process involving logarithm to solve $e^{0.021t} = 2$</p> <p>Obtain 33</p>	M1	with t the only variable; at least as far as $0.021t = \ln 2$; must be $\dots = 2$
	<p>State (or calculate separately to obtain) 99</p>	A1	or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0
		B1√	following previous answer; no need to include units
		[3]	
(ii)	<p>Differentiate to obtain $ke^{0.021t}$</p> <p>Obtain $250 \times 0.021 e^{0.021t}$</p> <p>Substitute to obtain 8.4 or $\frac{42}{5}$</p>	M1	where k is any constant not equal to 250
		A1	or simplified equiv $5.25e^{0.021t}$
		A1	or value rounding to 8.4 with no obvious error
		[3]	

Q10, (Jun 2014, Q1)

<p>Attempt use of product rule to find first derivative</p> <p>Obtain $8x \ln x + 4x$</p> <p>Attempt use of correct product rule to find second derivative</p> <p>Obtain $8 \ln x + 12$</p> <p>Obtain 28</p>	M1	producing form $\dots \pm \dots$ where one term involves $\ln x$ and the other does not
	A1	or unsimplified equiv
	M1	with one term involving $\ln x$
	A1	or unsimplified equiv
	A1	
	[5]	

Q11, (Jun 2015, Q1)

<p>Attempt use of quotient rule or, after adjustment, product rule</p> <p>Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv</p> <p>Substitute 2 to obtain -13 or equiv</p> <p>Attempt to find equation of tangent</p> <p>Obtain $y = -13x + 19$ or $13x + y - 19 = 0$</p>	*M1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x-8$ in denominator; allow M1 for numerator the wrong way round	For product rule attempt, *M1 for $k_1(3x-8)^{-1} + k_2(5x+4)(3x-8)^{-2}$ form and A1 for correct constants 5 and -3;
	A1	Allow if missing brackets implied by subsequent simplification or calculation	
	A1		
	M1	Dep *M; equation of tangent not normal	
	A1	Or similarly simplified equiv with 3 non-zero terms	
	[5]		

Q12, (Jun 2016, Q1)

Differentiate to produce form $k_1x(x+2)^m + k_2x^2(x+2)^n$	*M1	For positive integers k_1, k_2, m, n ; allow M1 if slip to, for example, $(x+3)$ in both brackets
Obtain $6x(x+2)^6 + 18x^2(x+2)^5$	A1	Or unsimplified equiv
Substitute $x = -1$ to obtain value 12	A1	From correct work only
Attempt equation of tangent (not normal) through point $(-1, 3)$	M1	Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates
Obtain $y = 12x + 15$	A1 [5]	Answer required in $y = mx + c$ form

Q13, (OCR 4753, Jan 2009, Q2i)

$y = x \cos 2x$	M1	product rule
$\frac{dy}{dx} = -2x \sin 2x + \cos 2x$	B1	$d/dx (\cos 2x) = -2 \sin 2x$
	A1	oe cao
	[3]	

Q14, (OCR 4753, Jan 2012, Q1)

$y = x^2 \tan 2x$	M1	product rule	$u \times \text{their } v' + v \times \text{their } u'$ attempted
$\Rightarrow dy/dx = 2x^2 \sec^2 2x + 2x \tan 2x$	M1	$d/du(\tan u) = \sec^2 u$ soi	M0 if $d/dx (\tan 2x) = (2) \sec^2 x$
OR $y = x^2 \frac{\sin 2x}{\cos 2x}$	A1 cao	or $2x^2/\cos^2 2x + 2x \tan 2x$	isw
$\frac{dy}{dx} = x^2 \frac{\cos 2x \cdot 2 \cos 2x - \sin 2x(-2 \sin 2x)}{\cos^2 2x} + 2x \frac{\sin 2x}{\cos 2x}$	M1	product rule	<i>see additional notes for complete solution</i>
$= \dots = 2x^2 \sec^2 2x + 2x \tan 2x$	A1	correct expression	$u \times \text{their } v' + v \times \text{their } u'$ attempted
OR $y = \frac{x^2 \sin 2x}{\cos 2x}$	A1 cao	or $2x^2/\cos^2 2x + 2x \tan 2x$ (isw)	or $(2x^2 + 2x \sin 2x \cos 2x)/\cos^2 2x$
$\frac{dy}{dx} = \frac{\cos 2x(2x \sin 2x + x^2 2 \cos 2x) - 2x^2 \sin 2x(-\sin 2x)}{\cos^2 2x}$	M1	quotient rule	or $2x^2/\cos^2 2x + 2x \sin 2x / \cos 2x$
$= \dots = 2x^2 \sec^2 2x + 2x \tan 2x$	A1 cao	correct expression	<i>see additional notes for complete solution</i>
	[3]	or $2x^2/\cos^2 2x + 2x \tan 2x$ (isw)	$(v \times \text{their } u' - u \times \text{their } v')/v^2$ attempted
			or $(2x^2 + 2x \sin 2x \cos 2x)/\cos^2 2x$
			or $2x^2/\cos^2 2x + 2x \sin 2x / \cos 2x$

Q15, (OCR 4753, Jun 2010, Q3)

(i) $y = (1+3x^2)^{1/2}$		
$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1+3x^2)^{-1/2} \cdot 6x$	M1	chain rule
$= 3x(1+3x^2)^{-1/2}$	B1	$\frac{1}{2} u^{-1/2}$
	A1	o.e., but must be '3'
	[3]	
(ii) $y = x(1+3x^2)^{1/2}$		
$\Rightarrow \frac{dy}{dx} = x \cdot \frac{3x}{\sqrt{1+3x^2}} + 1 \cdot (1+3x^2)^{1/2}$	M1	product rule
$= \frac{3x^2 + 1 + 3x^2}{\sqrt{1+3x^2}}$	A1 ft	ft their dy/dx from (i)
$= \frac{1+6x^2}{\sqrt{1+3x^2}} *$	M1	common denominator or factoring
	E1	$(1+3x^2)^{-1/2}$
	[4]	www

