Connected Rates Of Change (From OCR 4723)

Q1, (Jan 2007, Q4)

(i) Given that $x = (4t+9)^{\frac{1}{2}}$ and $y = 6e^{\frac{1}{2}x+1}$, find expressions for $\frac{dx}{dt}$ and $\frac{dy}{dx}$. [4]

(ii) Hence find the value of $\frac{dy}{dt}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]

Q2, (Jan 2008, Q4)

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of $\frac{dV}{dh}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

Q3, (Jan 2010, Q 7a)

Leaking oil is forming a circular patch on the surface of the sea. The area of the patch is increasing at a rate of 250 square metres per hour. Find the rate at which the radius of the patch is increasing at the instant when the area of the patch is 1900 square metres. Give your answer correct to 2 significant figures. **[4]**

Q4, (Jan 2011, Q3)

A giant spherical balloon is being inflated in a theme park. The radius of the balloon is increasing at a rate of 12 cm per hour. Find the rate at which the surface area of the balloon is increasing at the instant when the radius is 150 cm. Give your answer in cm^2 per hour correct to 2 significant figures.

[Surface area of sphere = $4\pi r^2$.] **[3]**

Q5, (Jun 2012, Q6)

The volume, $V\,m^3$, of liquid in a container is given by

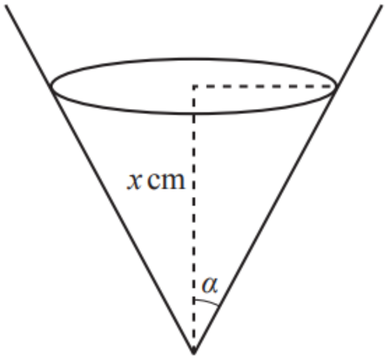
$$V = (3h^2 + 4)^{\frac{3}{2}} - 8\,,$$

where h m is the depth of the liquid.

(i) Find the value of $\frac{dV}{dh}$ when $h = 0.6$, giving your answer correct to 2 decimal places. **[4]**

(ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m, the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m. **[3]**

Q6, (Jun 2013, Q3)



The diagram shows a container in the form of a right circular cone. The angle between the axis and the slant height is α, where $\alpha = \tan^{-1}(\frac{1}{2})$. Initially the container is empty, and then liquid is added at the rate of 14 cm^3 per minute. The depth of liquid in the container at time t minutes is x cm.

(i) Show that the volume, V cm^3, of liquid in the container when the depth is x cm is given by

$$V = \frac{1}{12}\pi x^3.$$

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.] **[2]**

(ii) Find the rate at which the depth of the liquid is increasing at the instant when the depth is 8 cm. Give your answer in cm per minute correct to 2 decimal places. **[3]**

Q7, (Jun 2015, Q3)

The volume, V cubic metres, of water in a reservoir is given by

$$V = 3(2 + \sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. **[5]**

Q8, (Jun 2018, Q6a)

A reservoir is being filled with water at a constant rate of 15 cubic metres per minute. At the instant when the depth of the water is x metres, the volume of water in the reservoir is V cubic metres where

$$V = 2(5 + 2x)^3 - 250.$$

Find the rate at which the depth of the water is increasing at the instant when $x = 1.6$. **[4]**
