

Compound Angle Formulae – Rcos(x) and Rsin(x) (From OCR 4723)

Q1, (Jun 2006, Q8)

- (i) Express $5 \cos x + 12 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5 \cos x + 12 \sin x$. [3]
- (iii) Solve, for $0^\circ < x < 360^\circ$, the equation $5 \cos x + 12 \sin x = 2$, giving your answers correct to the nearest 0.1° . [5]

Q2, (Jan 2011, Q4)

- (i) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence solve the equation $24 \sin \theta + 7 \cos \theta = 12$ for $0^\circ < \theta < 360^\circ$. [4]

Q3, (Jun 2012, Q8)

- (i) Express $3 \sin \theta + 4 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence
- (a) solve the equation $3 \sin \theta + 4 \cos \theta + 1 = 0$, giving all solutions for which $-180^\circ < \theta < 180^\circ$, [4]
- (b) find the values of the positive constants k and c such that
- $$-37 \leq k(3 \sin \theta + 4 \cos \theta) + c \leq 43$$
- for all values of θ . [4]

Q4, (Jun 2013, Q8)

- (i) Express $4 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence
- (a) solve the equation $4 \cos \theta - 2 \sin \theta = 3$ for $0^\circ < \theta < 360^\circ$, [4]
- (b) determine the greatest and least values of
- $$25 - (4 \cos \theta - 2 \sin \theta)^2$$
- as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs. [5]

Q5, (Jun 2014, Q9)

- (i) Express $5 \cos(\theta - 60^\circ) + 3 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]
- (ii) Hence
- (a) give details of the transformations needed to transform the curve $y = 5 \cos(\theta - 60^\circ) + 3 \cos \theta$ to the curve $y = \sin \theta$, [3]
- (b) find the smallest positive value of β satisfying the equation
- $$5 \cos\left(\frac{1}{3}\beta - 40^\circ\right) + 3 \cos\left(\frac{1}{3}\beta + 20^\circ\right) = 3. [5]$$