Compound Angle Formulae – Rcos(x) and Rsin(x) (From OCR 4723)

Q1, (Jun 2006, Q8)

(i)	State $R = 13$	**B1**		or equiv
	State at least one equation of form $R\cos\alpha = k$, $R\sin\alpha = k'$, $\tan\alpha = k''$	**M1**		or equiv; allow sin / cos muddles; implied by correct α
	Obtain 67.4	**A1**	**3**	allow 67 or greater accuracy
(ii)	Refer to translation and stretch	**M1**		in either order; allow here equiv terms such as 'move', 'shift'; with both transformations involving constants
	State translation in positive x direction by 67.4	**A1√**		or equiv; following their α; using correct terminology now
	State stretch in y direction by factor 13	**A1√**	**3**	or equiv; following their R; using correct terminology now
iii)	Attempt value of $\cos^{-1}(2 \div R)$	**M1**		
	Obtain 81.15	**A1√**		following their R; accept 81
	Obtain 148.5 as one solution	**A1**		accept 148.5 or 148.6 or value rounding to either of these
	Add their α value to second value correctly attempted	**M1**		
	Obtain 346.2	**A1**	**5**	accept 346.2 or 346.3 or value rounding to either of these; and no other solutions

Q2, (Jan 2011, Q4)

(i)	Obtain $R = 25$	B1		allow $\sqrt{625}$ or value rounding to 25
	Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin\alpha = 7$, $\cos\alpha = 24$ in the working
	Obtain 16.3°	A1	**3**	or greater accuracy 16.260…; must be degrees now; allow 16° here
(ii)	Show correct process for finding one answer	M1		even if leading to answer outside 0 to 360
	Obtain (28.69 – 16.26 and hence) 12.4°	A1		or greater accuracy 12.425… or anything rounding to 12.4
	Show correct process for finding second answer	M1		even if further incorrect answers produced
	Obtain (151.31 – 16.26 and hence) 135° or 135.1°	A1	**4**	or greater accuracy 135.054…; and no other between 0 and 360
	[SC: No working shown and 2 correct angles stated - B1 only in part (ii)]			

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Q3, (Jun 2012, Q8)

(i)		State $R = 5$	B1	
		Attempt to find value of α	M1	implied by correct value or its complement
		Obtain 53.1	A1	allow $\tan^{-1}\frac{4}{3}$
			[3]	
(ii)	(a)	Attempt to find at least one value of $\theta + \alpha$	M1	(should be –168.5 or –11.5 or 191.5 or …)
		Obtain 1 correct value of θ (–64.7 or 138)	A1	allow ±0.1 in answer and greater accuracy
		Attempt correct process to find the second value	M1	involving a positive value of $\sin^{-1}(-\frac{1}{5})$ and subtraction of their α
		Obtain second value of θ (138 or –64.7)	A1	allow ±0.1 in answer and greater accuracy; and no others between –180 and 180
			[4]	
(ii)	(b)	Use –1 as minimum or 1 as maximum value of $\sin(\theta + \alpha)$	*M1	
		Relate $-5k + c$ to -37 and $5k + c$ to 43	A1	as equations or inequalities
		Attempt solution of pair of linear eqns	M1	dep *M; must be equations now
		Obtain $k = 8$ and $c = 3$	A1	**SC**: both $k = 8$ and $c = 3$ obtained with no working or from unconvincing working, award B2 (i.e. max 2/4)
			[4]	

Q4, (Jun 2013, Q8)

(i)		Obtain $R = \sqrt{20}$ or $R = 4.47$	B1	
		Attempt to find value of α	M1	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos\alpha = 4, \sin\alpha = 2$ here but not for A1
		Obtain 26.6	A1	or greater accuracy 26.565…; with no wrong working seen
			[3]	
(ii)	**(a)**	Show correct process for finding one answer	M1	allowing for case where the answer is negative
		Obtain 21.3	A1FT	or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R
		Show correct process for finding second answer	M1	ie attempting fourth quadrant value minus α value
		Obtain 286 or 285.6	A1FT	or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R ;and no others between 0° and 360°
			[4]	
(ii)	**(b)**	State greatest value is 25	B1	allow if α incorrect
		Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α
		State least value is 5	B1	allow if α incorrect
		Attempt to find θ from $\cos(\theta + \text{their}\alpha) = -1$	M1	and clearly associated with correct least value
		Obtain 153 or 153.4	A1FT	or greater accuracy 153.4349…; following a wrong value of α
			[5]	

Q5, (Jun 2014, Q9)

(i)		Simplify to obtain $\frac{11}{2}\cos\theta+\frac{5\sqrt{3}}{2}\sin\theta$	B1	or equiv with two terms perhaps with $\sin 60$ retained
		Attempt correct process to find R	M1	for expression of form $a\cos\theta+b\sin\theta$
		Attempt correct process to find α	M1	for expression of form $a\cos\theta+b\sin\theta$; condone $\sin\alpha=\frac{11}{2}$, $\cos\alpha=\frac{5}{2}\sqrt{3}$
		Obtain $7\sin(\theta+51.8)$	A1	or greater accuracy 51.786...
			[4]	
(ii)	**(a)**	State stretch and translation in either order	M1	or equiv but using correct terminology, not move, squash, ...
		State stretch parallel to y-axis with factor $\frac{1}{7}$	A1ft	following their R and clearly indicating correct direction
		State translation parallel to θ-axis or x-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix}51.8\\0\end{pmatrix}$	A1ft	following their α and clearly indicating correct direction; or equiv such as 308.2 parallel to x-axis in negative direction
			[3]	
	(b)	State left-hand side (their R) $\sin(\frac{1}{3}\beta+\gamma)$ where $\gamma\neq\pm$(their α), $\gamma\neq\pm40$, $\gamma\neq\pm20$	M1	or equiv such as stating $\theta=\frac{1}{3}\beta+20$
		Obtain (their R) $\sin(\frac{1}{3}\beta+$ their $\alpha+20)=3$	A1ft	(and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)
		Attempt correct process to find any value of $\frac{1}{3}\beta$	M1	for equation of form $\sin(\frac{1}{3}\beta+\gamma)=k$ where $\lvert k\rvert<1$, $k\neq0$
		Attempt complete process to find positive value of β	M1	including choosing second quadrant value of their $\sin^{-1}\frac{3}{7}$
		Obtain 248 or 249 or 248.5	A1	or greater accuracy 248.508...
			[5]	