Compound Angle Formulae Exam Questions (From OCR 4723)

Q1, (Jan 2009, Q9)

(i)	State $\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	B1		
	Use at least one of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
	Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2\theta$
	Obtain $4\cos^3\theta - 3\cos\theta$	A1	**4**	AG; necessary detail required

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(ii)	Either:	State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1		
		Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**3**	AG; necessary detail required
	Or:	State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1		maybe implied
		Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**(3)**	AG; necessary detail required

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(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
	Obtain $32c^6 - 48c^4 = 0$	A1		or equiv
	Attempt solution for c of equation	M1		dep *M
	Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
	Obtain $c = 0$, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only

Q2, (Jan 2010, Q9)

(i)	Identify $\tan 55°$ as $\tan(45° + 10°)$	B1		or equiv
	Use correct angle sum formula for $\tan(A+B)$	M1		or equiv
	Obtain $\dfrac{1+p}{1-p}$	A1	**3**	with $\tan 45°$ replaced by 1

(ii)	Either:	Attempt use of identity for $\tan 2A$	*M1		linking 10° and 5°
		Obtain $p = \dfrac{2t}{1-t^2}$	A1		
		Attempt solution for t of quadratic equation	M1		dep *M
		Obtain $\dfrac{-1+\sqrt{1+p^2}}{p}$	A1	**4**	or equiv; and no second expression
	Or (1):	Attempt expansion of $\tan(60° - 55°)$	*M1		
		Obtain $\dfrac{\sqrt{3} - \frac{1+p}{1-p}}{1+\sqrt{3}\,\frac{1+p}{1-p}}$	A1√		follow their answer from (i)
		Attempt simplification to remove denominators	M1		dep *M
		Obtain $\dfrac{\sqrt{3}(1-p)-(1+p)}{1-p+\sqrt{3}(1+p)}$	A1	**(4)**	or equiv

Or (2): State or imply $\tan 15° = 2-\sqrt{3}$	B1	
Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
Obtain $\dfrac{2-\sqrt{3}-p}{1+p(2-\sqrt{3})}$	A2 (4)	
Or (3): State or imply $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1	or exact equiv
Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
Obtain $\dfrac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 (4)	or equiv
Or (4): Attempt expansion of $\tan(10°-5°)$	*M1	
Obtain $t = \dfrac{p-t}{1+pt}$	A1	
Attempt solution for t of quadratic equation	M1	dep *M
Obtain $\dfrac{-2+\sqrt{4+4p^2}}{2p}$	A1 (4)	or equiv; and no second expression
(iii) Attempt expansion of both sides	M1	
Obtain $3\sin\theta\cos 10° + 3\cos\theta\sin 10° = 7\cos\theta\cos 10° + 7\sin\theta\sin 10°$	A1	or equiv
Attempt division throughout by $\cos\theta\cos 10°$	M1	or by $\cos\theta$ (or $\cos 10°$) only
Obtain $3t+3p = 7+7pt$	A1	or equiv
Obtain $\dfrac{3p-7}{7p-3}$	A1 **5**	or equiv
	12	

Q3, (Jun 2011, Q9)

(i)	Use at least one identity correctly	B1		angle-sum or angle-difference identity
	Attempt use of relevant identities in single rational expression	M1		not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F}=\frac{A}{D}+\frac{B}{E}+\frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha-\sin\theta\sin\alpha+3\cos\theta+\cos\theta\cos\alpha+\sin\theta\sin\alpha$)
	Obtain $\frac{2\sin\theta\cos\alpha+3\sin\theta}{2\cos\theta\cos\alpha+3\cos\theta}$	A1		or equiv but with the other two terms from each of num'r and den'r absent
	Attempt factorisation of num'r and den'r	M1		
	Obtain $\frac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$	A1	**5**	AG; necessary detail needed
(ii)	State or imply form $k\tan 150°$	M1		obtained without any wrong method seen
	State or imply $\frac{4}{3}\tan 150°$	A1		or equiv such as $\frac{12\sin 150°}{9\cos 150°}$
	Obtain $-\frac{4}{9}\sqrt{3}$	A1	**3**	or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3
(iii)	State or imply $\tan 6\theta = k$	B1		
	State $\frac{1}{6}\tan^{-1}k$	B1		
	Attempt second value of θ	M1		using $6\theta=\tan^{-1}k+$ (multiple of 180)
	Obtain $\frac{1}{6}\tan^{-1}k+30°$	A1	**4**	and no other value
			12	

Q4, (Jan 2013, Q9)

(i)		State $\cos\theta\cos 45-\sin\theta\sin 45$	B1	or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$
		Use correct identity for $\sin 2\theta$ or $\cos 2\theta$	B1	must be used; not earned for just a separate statement
		Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors, and showing two terms involving $\sin\theta\cos\theta$
		Obtain $\sin^2\theta$	A1	AG; necessary detail needed
			[4]	
(ii)		Use identity to produce equation of form $\sin\frac{1}{2}\theta = c$	M1	condoning single value of constant c here (including values outside the range -1 to 1); M0 for $\sin\theta = c$ unless value(s) are subsequently doubled
		Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…
		Obtain –70.5 or –70.6	A1√	or greater accuracy –70.528…; following first answer; and no other answer between –90 and 90; answer(s) only : 0/3
			[3]	
(iii)		State or imply $6\sin^2\frac{1}{3}\theta = k$	B1	
		Attempt to relate k to at least $6\sin^2 30°$	M1	
		Obtain $0 < k < \frac{3}{2}$	A1	condone use of $\le$
			[3]	

Q5, (Jun 2015, Q9)

(i)		Use at least one addition formula accurately	M1	Without substituting values for $\cos 30°$, etc. yet
		Obtain $\cos\theta$	A1	AG; necessary detail needed
		State $\cos 4\theta = 2\cos^2 2\theta - 1$	B1	Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$
		Attempt correct use of relevant formulae to express in terms of $\cos\theta$	M1	Or in terms of $\cos\theta$ and $\sin\theta$
		Obtain correct unsimplified expression in terms of $\cos\theta$ only	A1	e.g. $2(2c^2-1)^2 - 1 + 4(2c^2-1)$
		Simplify to confirm $8\cos^4\theta - 3$	A1	AG; necessary detail needed
			[6]	
(ii)	**(a)**	Obtain $\frac{1}{12}$	B1	
		Substitute 0 for $\cos\theta$ in correct expression	M1	No need to specify greatest and least
		Obtain $\frac{1}{4}$	A1	
			[3]	
	(b)	State or imply $8\cos^4(3\alpha) - 3 = 1$	B1	Or $2\cos^2 6\alpha + 4\cos 6\alpha - 2 = 0$
		Attempt correct method to obtain at least one value of α	M1	Allow for equation of form $\cos^4(3\alpha) = k$ where $0 < k < 1$ or for three-term quadratic equation in $\cos 6\alpha$
		Obtain 10.9	A1	Or greater accuracy 10.921…
		Obtain 49.1	A1	Or greater accuracy 49.078…; and no others between 0 and 60
			[4]	

Q6, (Jan 2006, Q9)

(i)	State $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	**B1**		
	Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$	**B1**		
	Attempt complete process to express in terms of $\sin\theta$	**M1**		[using correct identities]
	Obtain $3\sin\theta - 4\sin^3\theta$	**A1**	**4**	[**AG**; all correctly obtained]
(ii)	State 3	**B1**		
	Obtain expression involving $\sin 10\alpha$	**M1**		[allow θ/α confusion]
	Obtain 9	**A1**	**3**	[and no other value]
(iii)	Recognise cosec 2β as $\dfrac{1}{\sin 2\beta}$	**B1**		[allow θ/β confusion]
	Attempt to express equation in terms of $\sin 2\beta$ only	**M1**		[or equiv involving $\cos 2\beta$]
	Attempt to find non-zero value of $\sin 2\beta$	**M1**		[or of $\cos 2\beta$]
	Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$	**A1**		[or equiv, exact or approx]
	Attempt correct process to find two values of β	**M1**		[provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
	Obtain 20.1, 69.9	**A1**	**6**	[and no others between 0 and 90]

Q7, (Jun 2016, Q9)

i		Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1	
		State $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ or $\tan\theta + \dfrac{1}{\tan\theta}$	B1	Perhaps as part of expression
		Simplify using correct identities	M1	
		Obtain 2 correctly	A1	AG; necessary detail needed
			[4]	
ii	a	Obtain expression involving at least one of $\sin\frac{1}{6}\pi$ and $\sin\frac{1}{4}\pi$	M1	
		Obtain $\dfrac{2}{\sin\frac{1}{6}\pi} + \dfrac{2}{\sin\frac{1}{4}\pi}$	A1	Or equiv involving cosecant
		Obtain $4 + 2\sqrt{2}$ or exact equiv	A1	Answer only is 0/3
			[3]	
	b	Use $\sin 4\theta = 2\sin 2\theta \cos 2\theta$	B1	
		Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2\theta = \frac{5}{8}$ or $\sin^2\theta = \frac{3}{8}$	B1	
		Obtain 0.659 or 0.66	B1	Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π; answer only earns 0/3
			[3]	

c	Express in form $k_1 \sin^4\theta \times \frac{k_2}{\sin^3\theta}$	M1	
	Obtain $4\sin^4\theta \times \frac{8}{\sin^3\theta}$ and hence $32\sin\theta$	A1	A0 if $(-2\sin^2\theta)^2$ involved in simplification
		[2]	

Q8, Jun 2017, Q8)

i	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1	Must be used not merely stated
	Obtain $6\sin\theta + 8\cos\theta$	B1	May be implied
	Obtain $R = 10$	B1	From correct $6\sin\theta + 8\cos\theta$
	Attempt appropriate trigonometry to find α	M1	Allow for $\tan\alpha = \frac{6}{8}$ or equiv
	Obtain $53.1°$	A1	Or greater accuracy $53.13\ldots$; with no errors seen
		[5]	
ii	State or imply equation is $10\sin(\beta + 63.1°) = 3$	B1ft	Following their R and α
	Carry out correct process to find one value of β	M1	Not available for finding negative angle; must involve use of 2nd quadrant angle
	Obtain $99.4°$ (or $314°$)	A1	Or greater accuracy $99.4122\ldots°$
	Carry out correct process to find second value of β	M1	Must involve use of '5th' quadrant angle
	Obtain $314°$ (or $99.4°$)	A1	Accept value rounding to 314 providing no error; and no others between 0 and 360
			[Note: Solving $10\sin(\theta + 53.1°) = 3$ can earn M1 M1 if correct processes followed; if continue to find correct angles by subtracting $10°$, A1 A1 available; B1 can be retrospectively given even if answers are wrong]
		[5]	