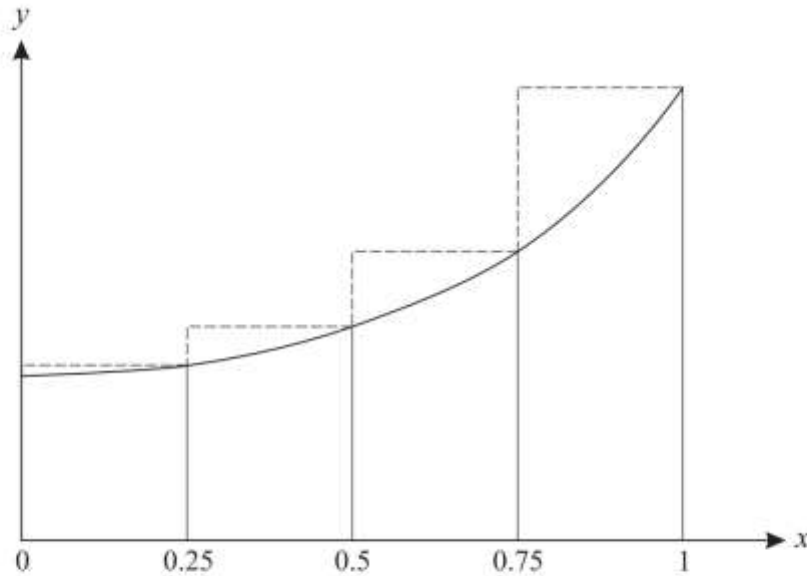


Bounds on Integrals Using Rectangles (From OCR 4726)

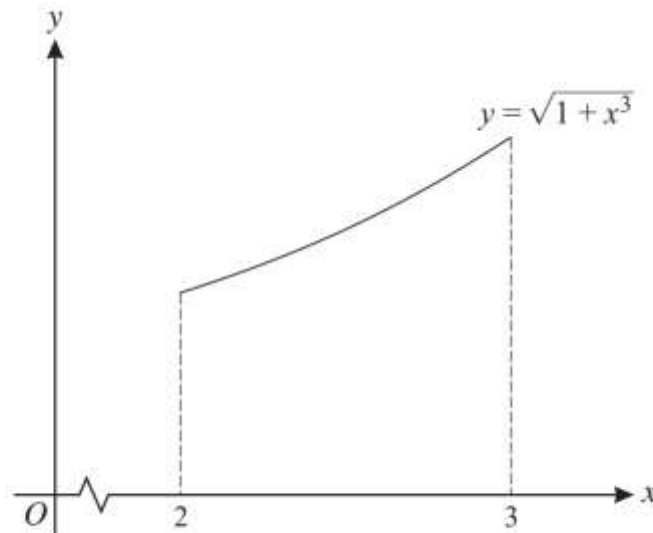
Q1, (Jan 2007, Q3)



The diagram shows the curve with equation $y = e^{x^2}$, for $0 \leq x \leq 1$. The region under the curve between these limits is divided into four strips of equal width. The area of this region under the curve is A .

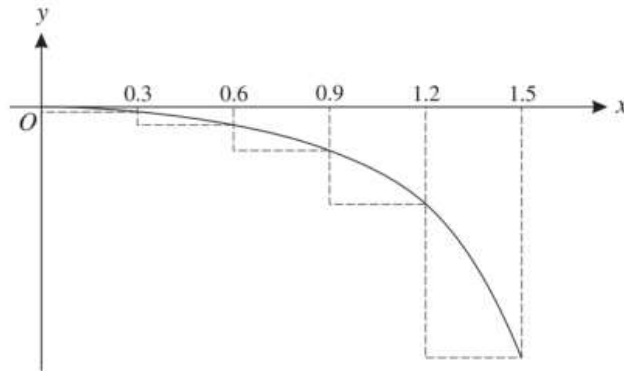
- (i) By considering the set of rectangles indicated in the diagram, show that an upper bound for A is 1.71. [3]
- (ii) By considering an appropriate set of four rectangles, find a lower bound for A . [3]

Q2, (Jan 2008, Q3)



The diagram shows the curve with equation $y = \sqrt{1 + x^3}$, for $2 \leq x \leq 3$. The region under the curve between these limits has area A .

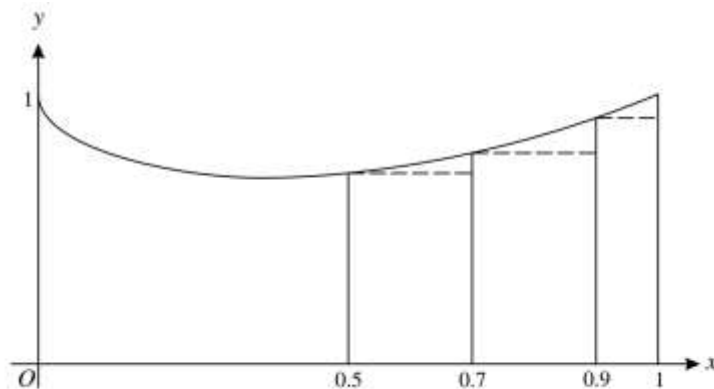
- (i) Explain why $3 < A < \sqrt{28}$. [2]
- (ii) The region is divided into 5 strips, each of width 0.2. By using suitable rectangles, find improved lower and upper bounds between which A lies. Give your answers correct to 3 significant figures. [4]



The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \leq x \leq 1.5$. The region bounded by the curve, the x -axis and the line $x = 1.5$ has area A . The region is divided into five strips, each of width 0.3.

- (i) By considering the set of rectangles indicated in the diagram, find an upper bound for A . Give the answer correct to 3 decimal places. [2]
- (ii) By considering another set of five suitable rectangles, find a lower bound for A . Give the answer correct to 3 decimal places. [2]
- (iii) How could you reduce the difference between the upper and lower bounds for A ? [1]

Q4, (Jan 2011, Q6)



The diagram shows the curve $y = f(x)$, defined by

$$f(x) = \begin{cases} x^x & \text{for } 0 < x \leq 1, \\ 1 & \text{for } x = 0. \end{cases}$$

- (i) By first taking logarithms, show that the curve has a stationary point at $x = e^{-1}$. [3]

The area under the curve from $x = 0.5$ to $x = 1$ is denoted by A .

- (ii) By considering the set of three rectangles shown in the diagram, show that a lower bound for A is 0.388. [2]
- (iii) By considering another set of three rectangles, find an upper bound for A , giving 3 decimal places in your answer. [2]

The area under the curve from $x = 0$ to $x = 0.5$ is denoted by B .

- (iv) Draw a diagram to show rectangles which could be used to find lower and upper bounds for B , using not more than three rectangles for each bound. (You are not required to find the bounds.) [3]