**Bounds on Integrals Using Rectangles (From OCR 4726)**

**Q1, (Jan 2007, Q3)**

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|---|---|
| (i) Show area of rect. = $\frac{1}{4}(e^{1/16} + e^{1/4} + e^{9/16} + e^{1})$ | M1 Or numeric equivalent |
| Show area = 1.7054 | A1 At least 3 d.p. correct |
| Explain the < 1.71 in terms of areas | B1 AG. Inequality required |
| (ii) Identify areas for > sign | B1 Inequality or diagram required |
| Show area of rect. = $\frac{1}{4}(e^{0} + e^{1/16} + e^{1/4} + e^{9/16})$ | M1 Or numeric evidence |
| Get $A > 1.27$ | A1 cao; or answer which rounds down |

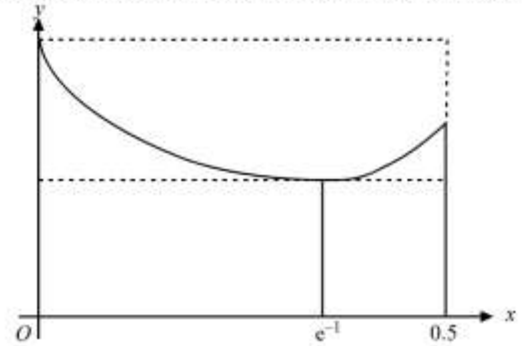
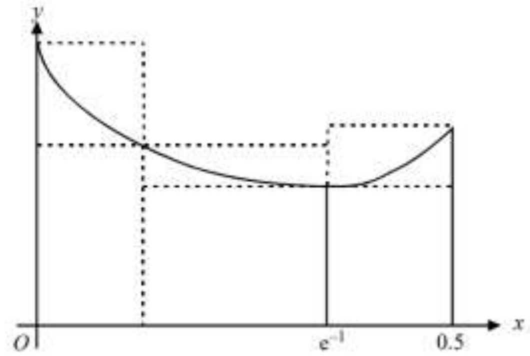
**Q2, (Jan 2008, Q3)**

| | | | |
|---|---|---|---|
| **(i)** | Get $y$- values of 3 and $\sqrt{28}$ | B1 | |
| | Show/explain areas of two rectangles equal $y$- value x 1, and relate to $A$ | B1 | Diagram may be used |
| **(ii)** | Show $A>0.2(\sqrt{(1+2^3)} + \sqrt{(1+2.2^3)} + \ldots \sqrt{(1+2.8^3)})$ | M1 | Clear areas attempted below curve (5 values) |
| | = 3.87(28) | A1 | To min. of 3 s.f. |
| | Show $A<0.2(\sqrt{(1+2.2^3)} + \sqrt{(1+2.4^3)} + \ldots + \sqrt{(1+3^3)})$ | M1 | Clear areas attempted above curve (5 values) |
| | = 4.33(11) <4.34 | A1 | To min. of 3 s.f. |

**Q3, (Jun 2009, Q1)**

| | | | |
|---|---|---|---|
| **(i)** | Attempt area = $\pm\Sigma(0.3y)$ for at least three $y$ values | M1 | May be implied |
| | Get 1.313(1..) or 1.314 | A1 | Or greater accuracy |
| **(ii)** | Attempt ± sum of areas (4 or 5 values) | M1 | May be implied |
| | Get 0.518(4..) | A1 | Or greater accuracy<br>**SC**<br>If answers only seen,<br>1.313(1..) or 1.314 B2<br>0.518(4..) B2<br>−1.313(1..) or −1.314 B1<br>−0.518(4..) B1 |
| | Or | | |
| | Attempt answer to part **(i)**−final rectangle | M1 | |
| | Get 0.518(4..) | A1 | |
| **(iii)** | Decrease width of strips | B1 | Use more strips or equivalent |

**Q4, (Jan 2011, Q6)**

| | | | | |
|---|---|---|---|---|
| (i) | $y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$ | M1 | | For differentiating $\ln y$ *OR* $x \ln x$ w.r.t. $x$ |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1+\ln x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = \mathrm{e}^{-1}$ | A1 | | For $(1+\ln x)$ seen or implied |
| | | A1 | **3** | For correct $x$-value from fully correct working **AG** |
| (ii) | $A > 0.2 \times 0.5^{0.5} + 0.2 \times 0.7^{0.7} + 0.1 \times 0.9^{0.9}$ | M1 | | For areas of 3 lower rectangles |
| | $\Rightarrow A > 0.3881(858) > 0.388$ | A1 | **2** | For lower bound rounding to **AG** |
| (iii) | $A < 0.2 \times 0.7^{0.7} + 0.2 \times 0.9^{0.9} + 0.1 \times 1^1$ | M1 | | For areas of 3 upper rectangles |
| | $\Rightarrow A < 0.4377(177) < 0.438$ | A1 | **2** | For upper bound rounding to 0.438 |
| (iv) |  | M1 | | Consider rectangle of height $\mathrm{f}(\mathrm{e}^{-1})$ |
| | | A1 | | Use at least 1 lower rectangle, height $\mathrm{f}(\mathrm{e}^{-1})$ |
| | | B1 | **3** | Use at least 1 upper rectangle, height $\mathrm{f}(0)$ |
| |  | | | **SR** If more than one rectangle is used for either bound, they must be shown correctly |



**10**