**Applied Sequences and Series Exam Questions (From OCR MEI 4752)**

**Q1, (Jun 2010, Q12)**

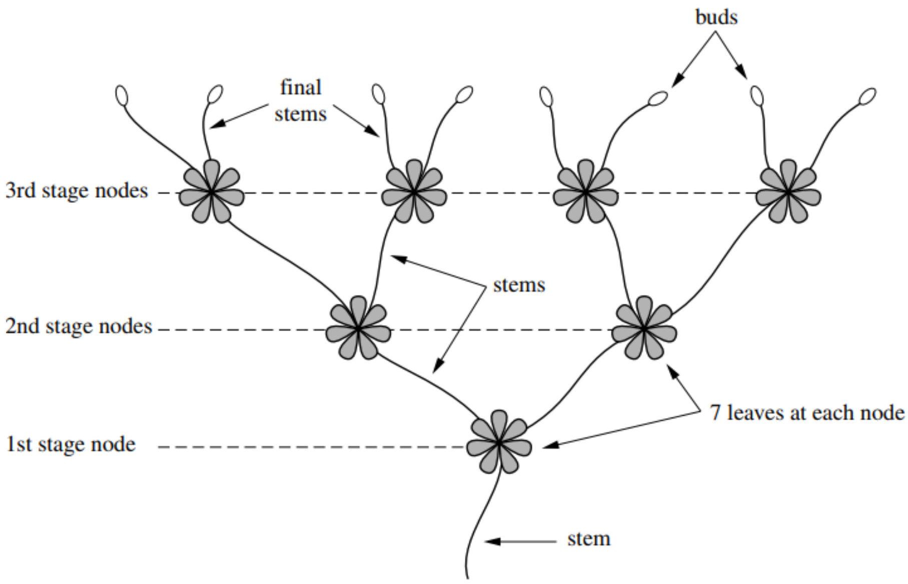


**Fig. 12**

A branching plant has stems, nodes, leaves and buds.

- There are 7 leaves at each node.
- From each node, 2 new stems grow.
- At the end of each final stem, there is a bud.

Fig. 12 shows one such plant with 3 stages of nodes. It has 15 stems, 7 nodes, 49 leaves and 8 buds.

**(i)** One of these plants has 10 stages of nodes.

($A$) How many buds does it have? **[2]**

($B$) How many stems does it have? **[2]**

**(ii)** ($A$) Show that the number of leaves on one of these plants with $n$ stages of nodes is

$$7(2^n - 1).$$ **[2]**

($B$) One of these plants has $n$ stages of nodes and more than 200 000 leaves. Show that $n$ satisfies the inequality $n > \dfrac{\log_{10} 200\,007 - \log_{10} 7}{\log_{10} 2}$. Hence find the least possible value of $n$.

**[4]**

**Q2, (Jun 2005, Q11)**

There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.

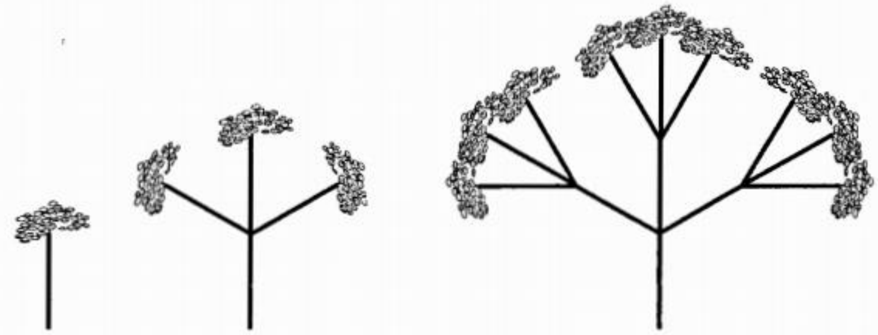


**Fig. 11**

**(i)** How many flowerheads are there in year 5? [1]

**(ii)** How many flowerheads are there in year $n$? [1]

**(iii)** As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, $1 + 3 + 9$).

Show that the total number of stems in year $n$ is given by $\dfrac{3^n - 1}{2}$. [2]

**(iv)** Kitty's oleander has a total of 364 stems. Find

(*A*) its age, [2]

(*B*) how many flowerheads it has. [1]

**(v)** Abdul's oleander has over 900 flowerheads.

Show that its age, $y$ years, satisfies the inequality $y > \dfrac{\log_{10}900}{\log_{10}3} + 1$.

Find the smallest integer value of $y$ for which this is true. [4]

**Q3, (Jun 2007, Q11)**

**(a)** André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

**(i)** How many counters are there in his sixth pile? [1]

**(ii)** André makes ten piles of counters. How many counters has he used altogether? [2]

**(b)** In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability $P_n$ of Betty starting on her $n$th throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

**(i)** Calculate $P_4$. Give your answer as a fraction. [2]

**(ii)** The values $P_1, P_2, P_3, \ldots$ form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \ldots = 1$. [3]

**(iii)** Given that $P_n < 0.001$, show that $n$ satisfies the inequality

$$n > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)} + 1.$$

Hence find the least value of $n$ for which $P_n < 0.001$. [4]

**Q4, (Jun 2011, Q12)**

Jim and Mary are each planning monthly repayments for money they want to borrow.

**(i)** Jim's first payment is £500, and he plans to pay £10 less each month, so that his second payment is £490, his third is £480, and so on.

($A$) Calculate his 12th payment. **[2]**

($B$) He plans to make 24 payments altogether. Show that he pays £9240 in total. **[2]**

**(ii)** Mary's first payment is £460 and she plans to pay 2% less each month than the previous month, so that her second payment is £450.80, her third is £441.784, and so on.

($A$) Calculate her 12th payment. **[2]**

($B$) Show that Jim's 20th payment is less than Mary's 20th payment but that his 19th is not less than her 19th. **[3]**

($C$) Mary plans to make 24 payments altogether. Calculate how much she pays in total. **[2]**

($D$) How much would Mary's first payment need to be if she wishes to pay 2% less each month as before, but to pay the same in total as Jim, £9240, over the 24 months? **[2]**

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**Q5, (Jun 2009, Q11)**

**(i)** In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.

($A$) Haroon answers six questions correctly. Show that he receives a total of £210. **[1]**

($B$) State, in a simple form, a formula for the total amount received by a contestant who answers $n$ questions correctly.

Hence find the value of $n$ for a contestant who receives £10 350 from this game. **[4]**

**(ii)** In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.

($A$) Gary received £75 from the game. How many questions did he get right? **[1]**

($B$) Bethan answered 9 questions correctly. How much did she receive from the game? **[2]**

($C$) State a formula for the total amount received by a contestant who answers $n$ questions correctly.

Hence find the value of $n$ for a contestant in this game who receives £2 621 435. **[4]**

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**Q6, (Jun 2015, Q11)**

Jill has 3 daughters and no sons. They are generation 1 of Jill's descendants.

Each of her daughters has 3 daughters and no sons. Jill's 9 granddaughters are generation 2 of her descendants. Each of her granddaughters has 3 daughters and no sons; they are descendant generation 3.

Jill decides to investigate what would happen if this pattern continues, with each descendant having 3 daughters and no sons.

**(i)** How many of Jill's descendants would there be in generation 8? **[2]**

**(ii)** How many of Jill's descendants would there be altogether in the first 15 generations? **[3]**

**(iii)** After $n$ generations, Jill would have over a million descendants altogether. Show that $n$ satisfies the inequality

$$n > \frac{\log_{10} 2000003}{\log_{10} 3} - 1.$$

Hence find the least possible value of $n$. **[4]**

**(iv)** How many **fewer** descendants would Jill have altogether in 15 generations if instead of having 3 daughters, she and each subsequent descendant has 2 daughters? **[3]**