Variable Acceleration in Two Dimensions (From OCR 4761)

Q1, (Jan 2005, Q1)

(i)	Differentiate $\mathbf{v} = 2t\,\mathbf{i} + (5 - 4t)\,\mathbf{j}$	M1 A1	At least 1 cpt correct Award for RHS seen	
	Differentiate $\mathbf{a} = 2\,\mathbf{i} - 4\,\mathbf{j}$	M1 F1	Do not award if **i** and **j** lost in **v.** At least 1 cpt correct. FT FT from **their** 2 component **v**	4
(ii)	$\mathbf{F} + 12\,\mathbf{j} = 4(2\,\mathbf{i} - 4\,\mathbf{j})$	M1 A1	N2L. Allow $\mathbf{F} = m\mathbf{a}$. No extra forces. Allow 12**j** omitted Allow wrong signs otherwise correct with **their** vector **a**.	
	$\mathbf{F} = 8\,\mathbf{i}$ Ń28 $\mathbf{j}$	A1	cao	3
	total	7		

Q2, (Jun 2005, Q5)

(i)	$x = 2 \Rightarrow t = 4$ $t = 4 \Rightarrow y = 16 - 1 = 15$	B1 F1	cao FT **their** t and y. Accept 15 **j**	2
(ii)	$x = \frac{1}{2}t$ and $y = t^2 - 1$ Eliminating t gives $y = \left((2x)^2 - 1\right) = 4x^2 - 1$	M1 E1	Attempt at elimination of expressions for x and y in terms of t Accept seeing $(2x)^2\text{-}1 = 4x^2 - 1$	2
(iii)	**either** We require $\frac{dy}{dx} = 1$ so $8x = 1$ $x = \frac{1}{8}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	M1 B1 A1	This may be implied Differentiating correctly to obtain $8x$	3
	or Differentiate to find **v** equate **i** and **j** cpts so $t = \frac{1}{4}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	M1 M1 A1	 Equating the **i** and **j** cpts of **their v**	
	total	7		

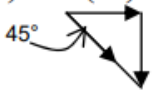
Q3, (Jun 2006, Q4)

(i)	**either**			
	Need **j** cpt 0 so $18t^2 - 1 = 0$	M1	Need not solve	
	$\Rightarrow t^2 = \frac{1}{18}$. Only one root as $t > 0$	E1	Must establish only one of the two roots is valid	
	or			
	Establish sign change in **j** cpt	B1		
	Establish only one root	B1		
				2
(ii)	$\mathbf{v} = 3\,\mathbf{i} + 36t\,\mathbf{j}$	M1	Differentiate. Allow **i** or **j** omitted	
		A1		
	Need **i** cpt 0 and this never happens	E1	Clear explanation. Accept '**i** cpt always there' or equiv	
				3
(iii)	$x = 3t$ and $y = 18t^2 - 1$	B1	Award for these two expressions seen.	
	Eliminate t to give			
	$y = 18\left(\frac{x}{3}\right)^2 - 1$	M1	t properly eliminated. Accept any form and brackets missing	
	so $y = 2x^2 - 1$	A1	cao	
				3
				8

Q4, (Jan 2006, Q5)

(i)	$9\mathbf{i}$ m s^{-2}; $(9\mathbf{i} - 12\mathbf{j})$ m s^{-2}	B1	Award for either. Accept no units. (isw e.g. finding magnitudes)	1
(ii)	N2L $\mathbf{F} = 4\,(9\mathbf{i} - 12\mathbf{j}) = (36\mathbf{i} - 48\,\mathbf{j})$ N	B1	Accept factored form. isw. FT **a**(3). Accept 60 N or **their** $4\lvert\mathbf{a}\rvert$	1
(iii)	$\mathbf{v} = \int \begin{pmatrix} 9 \\ -4t \end{pmatrix} dt = \begin{pmatrix} 9t + C \\ -2t^2 + D \end{pmatrix}$	M1	Integration. At least one term correct.	
		A1	Neglect arbitrary constant(s)	
	Using $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$ when $t = 1$ $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 + C \\ -2 + D \end{pmatrix}$	M1	Sub at $t = 1$ to find arb const(s)	
	$\Rightarrow C = -5, D = 4$ so $\mathbf{v} = (9t - 5)\mathbf{i} + (4 - 2t^2)\mathbf{j}$	A1	Any form	4
				6

Q5, (Jan 2007, Q6)

(i)	$t = 2.5 \Rightarrow \mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + 2.5\begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$  speed is $\sqrt{10^2 + 10^2} = 14.14...$ so 14.1 m s^{-1} (3 s. f.)	B1 E1 F1	Need not be in vector form Accept diag and/or correct derivation of just ±45° FT **their v**	3
(ii)	$\mathbf{s} = 2.5\begin{pmatrix} -5 \\ 10 \end{pmatrix} + \frac{1}{2} \times 2.5^2 \times \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} 6.25 \\ 0 \end{pmatrix}$ so 090°	M1 A1 A1 A1	Consideration of **s** (const accn or integration) Correct sub into *uvast* with **u** and **a.** (If integration used it must be correct but allow no arb constant) cao. CWO.	4
				7

Q6, (Jun 2009, Q5)

(i)

$\mathbf{v} = \mathbf{i} + (3 - 2t)\mathbf{j}$	M1 A1	Differentiating **r.** Allow 1 error. Could use const accn.	
$\mathbf{v}(4) = \mathbf{i} - 5\mathbf{j}$	F1	Do not award if $\sqrt{26}$ is given as vel (accept if **v** given and *v* given as well called speed or magnitude).	3

(ii)

$\mathbf{a} = -2\mathbf{j}$	B1	Diff **v.** FT **their v.** Award if $-2\mathbf{j}$ seen & isw.	
Using N2L $\mathbf{F} = 1.5 \times (-2\mathbf{j})$	M1	Award for $1.5 \times (\pm$ **their a** or $a)$ seen.	
so $-3\mathbf{j}$ N	A1	cao Do not award if final answer is not correct. [Award M1 A1 for $-3\mathbf{j}$ WW]	3

(iii)

$x = 2 + t$ and $y = 3t - t^2$	B1	Must have both but may be implied.	
Substitute $t = x - 2$ so $y = 3(x-2) - (x-2)^2$ $[= (x-2)(5-x)]$	B1	cao. isw. Must see the form $y =$ ….	2

Q7, (Jun 2013, Q4)

Equate **i** and **j** components of **v**	M1	The candidate recognises that the **i** and **j** components must be equal.
$16-t^2=31-8t$	A1	An equation is formed.
$t^2-8t+15=0$		
$(t-3)(t-5)=0$		
$t=3$ or 5	A1	May be implied by later working.
When $t=3$, $\mathbf{v}=7\mathbf{i}+7\mathbf{j}$	B1	
Speed when $t=3$ is $7\sqrt{2}=9.9\ \text{m s}^{-1}$	B1	
The values of the **i** and **j** components must both be positive for the bearing to be 045°.	B1	This mark is dependent on obtaining A1 for the result $t=3$ or 5. It is awarded if the speed for the case when $t=5$ is not included (since $t=5 \Rightarrow \mathbf{v}=-9\mathbf{i}-9\mathbf{j}$ and the bearing is 225°).
		Note Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.
	[6]	

Q8, (Jan 2011, Q4)

(i)	When $t = 1$, $\mathbf{r} = \begin{pmatrix} 8 \\ 10-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$		Accept column or $a\mathbf{i} + b\mathbf{j}$ notation
	$[8\mathbf{i} + (10-2)\mathbf{j} = 8\mathbf{i} + 8\mathbf{j}]$	B1	May be implied
	Bearing OP is 045°	F1	Accept 45°. Accept NE and northeast. Condone $\lvert\mathbf{r}\rvert$ given as well.
		2	
(ii)	$\mathbf{v} = \begin{pmatrix} 8 \\ 20t - 6t^2 \end{pmatrix}$ $[8\mathbf{i} + (20t - 6t^2)\mathbf{j}]$	M1	Differentiating both components. Condone 1 error if clearly attempting differentiation.
		A1	Must be a vector answer.
	The **i** cpt is always 8 so $\mathbf{v} \neq \mathbf{0}$ for any t	E1	Accept any correct argument e.g. based on **i** cpt never 0.
		3	
(iii)	$\mathbf{a} = \begin{pmatrix} 0 \\ 20 - 12t \end{pmatrix}$ $[(20 - 12t)\mathbf{j}]$	M1	Differentiating as a vector. Condone 1 error if clearly attempting differentiation of **their v**.
		F1	FT **their v**.
	$\mathbf{a} = \mathbf{0}$ when $t = \frac{20}{12} = \frac{5}{3}$		
	so $\frac{5}{3}$ s (1.67 s (3 s. f.))	B1	cao. Condone obtained from scalar equation.
		3	
		8	

Q9, (Jun 2015, Q3)

(i)		**Either** $-2+8t=7t$ **Or** $t=10-4t$	**M1**	Forming an equation for t. Accept vector equation for this mark. May be implied by a statement that $t=2$.
		$\Rightarrow t=2$	**A1**	
		Substituting $t=2$ in **both** expressions	**B1**	oe, eg showing $t=2$ satisfies both equations or a vector equation.
		They meet at (14, 2)	**B1**	Accept $\begin{pmatrix}14\\2\end{pmatrix}$
			[4]	
(ii)		Ashok's speed is $\sqrt{8^2+1^2}=\sqrt{65}$	**B1**	
		Kumar's speed is $\sqrt{7^2+(-4)^2}=\sqrt{65}$ km h^{-1}	**B1**	
		They both walk at the same speed	**B1**	CAO from correct speeds SC1 for finding both velocities correctly but neither speed
			[3]	

Q10, (Jun 2016, Q4)

(i)	$\mathbf{u} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$	**B1**	
	$\mathbf{a} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}$	**B1**	
		[2]	
(ii)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$		
	$t = 2 \Rightarrow \mathbf{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 \\ -8 \end{pmatrix} \times 2$	**M1**	Or equivalent. FT for their **u** and **a**
	$= \begin{pmatrix} 2 \\ -10 \end{pmatrix}$	**A1**	Continue the FT for this mark
	Speed $= \sqrt{2^2 + (-10)^2} = 10.2\,\text{m s}^{-1}$ (to 3 sf)	**B1**	FT from their **v**
		[3]	
(iii)	$x = ut \Rightarrow x = 2t \Rightarrow t = \frac{x}{2}$	**M1**	This mark may also be obtained for substituting x for $2t$ in the expression for y.
	$y = 6t - 4t^2$	**B1**	
	$y = 6 \times \frac{x}{2} - 4 \times \left(\frac{x}{2}\right)^2 = 3x - x^2$	**A1**	
		[3]	