Forces and Motion in 3 Dimensions (From OCR 4761)

Q1, (Jan 2005, Q3)

A particle is in equilibrium when acted on by the forces $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix}$, $\begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$, where the units are newtons.

(i) Find the values of x, y and z. [4]

(ii) Calculate the magnitude of $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$. [2]

Q2, (Jan 2008, Q4)

Force **F** is $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ N and force **G** is $\begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$ N.

(i) Find the resultant of **F** and **G** and calculate its magnitude. [4]

(ii) Forces **F**, 2**G** and **H** act on a particle which is in equilibrium. Find **H**. [3]

Q3, (Jun 2010, Q3)

The three forces $\begin{pmatrix} -1 \\ 14 \\ -8 \end{pmatrix}$ N, $\begin{pmatrix} 3 \\ -9 \\ 10 \end{pmatrix}$ N and **F** N act on a body of mass 4 kg in deep space and give it an acceleration of $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ m s^{-2}.

(i) Calculate **F**. **[4]**

At one instant the velocity of the body is $\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$ m s^{-1}.

(ii) Calculate the velocity and also the speed of the body 3 seconds later. **[4]**

Q4, (Jun 2011, Q3)

Force **F** is $\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$ N, force **G** is $\begin{pmatrix} -6 \\ y \\ z \end{pmatrix}$ N and force **H** is $\begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$ N.

(i) Given that **F** and **G** act in parallel lines, find y and z. **[2]**

Forces **F** and **H** are the only forces acting on an object of mass 5 kg.

(ii) Calculate the acceleration of the object. Calculate also the magnitude of this acceleration. **[5]**

Q5, (Jun 2014, Q6)

In this question the origin is a point on the ground. The directions of the unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ are east, north and vertically upwards.

Alesha does a sky-dive on a day when there is no wind. The dive starts when she steps out of a moving helicopter. The dive ends when she lands gently on the ground.

- During the dive Alesha can reduce the magnitude of her acceleration in the vertical direction by spreading her arms and increasing air resistance.
- During the dive she can use a power unit strapped to her back to give herself an acceleration in a horizontal direction.
- Alesha's mass, including her equipment, is 100 kg.
- Initially, her position vector is $\begin{pmatrix}-75\\90\\750\end{pmatrix}$ m and her velocity is $\begin{pmatrix}-5\\0\\-10\end{pmatrix}$ m s^{-1}.

At a certain time during the dive, forces of $\begin{pmatrix}0\\0\\-980\end{pmatrix}$ N, $\begin{pmatrix}0\\0\\880\end{pmatrix}$ N and $\begin{pmatrix}50\\-20\\0\end{pmatrix}$ N are acting on Alesha.

(ii) Suggest how these forces could arise. **[3]**

(iii) Find Alesha's acceleration at this time, giving your answer in vector form, and show that, correct to 3 significant figures, its magnitude is 1.14 m s^{-2}. **[3]**

One suggested model for Alesha's motion is that the forces on her are constant throughout the dive from when she leaves the helicopter until she reaches the ground.

(iv) Find expressions for her velocity and position vector at time t seconds after the start of the dive according to this model. Verify that when $t = 30$ she is at the origin. **[6]**

(v) Explain why consideration of Alesha's landing velocity shows this model to be unrealistic. **[2]**

Q6, (Jun 2013, Q3)

In this question take $g = 10$.

The directions of the unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ are east, north and vertically upwards.

Forces **p**, **q** and **r** are given by $\mathbf{p} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}$ N, $\mathbf{q} = \begin{pmatrix}-1\\-4\\2\end{pmatrix}$ N and $\mathbf{r} = \begin{pmatrix}2\\5\\0\end{pmatrix}$ N.

(i) Find which of **p**, **q** and **r** has the greatest magnitude. **[2]**

(ii) A particle has mass 0.4 kg. The forces acting on it are **p**, **q**, **r** and its weight.

Find the magnitude of the particle's acceleration and describe the direction of this acceleration. **[4]**