Constant Acceleration in Two Dimensions (From Edexcel 6677)

Q1, (Jun 2010, Q1)

$(-4\mathbf{i}-7\mathbf{j}) = \mathbf{r} + 4(-3\mathbf{i}+2\mathbf{j})$ M1 A1

$\mathbf{r} = (8\mathbf{i}-15\mathbf{j})$ A1

$|\mathbf{r}| = \sqrt{8^2+(-15)^2} = 17 \text{ m}$ M1 A1 ft

[5]

Q2, (Jan 2009, Q1)

$-6\mathbf{i}+\mathbf{j} = \mathbf{u} + 3(2\mathbf{i}-5\mathbf{j})$ M1 A1

$\Rightarrow \mathbf{u} = -12\mathbf{i}+16\mathbf{j}$ A1 cso

$\Rightarrow u = \sqrt{(-12)^2+16^2} = 20$ M1 A1

[5]

Q3, (Jan 2005, Q7)

(a) $\mathbf{v}_P = \{(29\mathbf{i}+34\mathbf{j}) - (20\mathbf{i}+10\mathbf{j})\}/3 = \underline{(3\mathbf{i}+8\mathbf{j}) \text{ km h}^{-1}}$ M1 A1

(2)

(b) $\mathbf{p} = (20\mathbf{i}+10\mathbf{j}) + (3\mathbf{i}+8\mathbf{j})t$ M1 A1√

$\mathbf{q} = (14\mathbf{i}-6\mathbf{j}) + 12t\mathbf{j}$ M1 A1

(4)

(c) $\mathbf{q}-\mathbf{p} = (-6-3t)\mathbf{i} + (-16+4t)\mathbf{j}$ M1 A1

↓

$d^2 = (-6-3t)^2 + (-16+4t)^2$ M1

↓

$= 36+36t+9t^2+16t^2-128t+256$ M1

$= 25t^2 - 92t + 292$ (*) A1 (cso)

(5)

(d) $25t^2 - 92t + 292 = 225$ M1

$25t^2 - 92t + 67 = 0$ A1

↓

$(t-1)(25t-67) = 0$ M1

$t = 67/25$ or 2.68 A1

time ≈ 161 mins, or 2 hrs 41 mins, or 2.41 am, or 0241 A1

(5)

Q4, (Jun 2006, Q7)

(a) Speed $= \sqrt{(2.5^2 + 6^2)} = \underline{6.5 \text{ km h}^{-1}}$ — M1 A1 (2)

(b) Bearing $= 360 - \arctan(2.5/6) \approx \underline{337}$ — M1 A1 (2)

(c) $\mathbf{R} = (16 - 3 \times 2.5)\mathbf{i} + (5 + 3 \times 6)\mathbf{j}$ — M1

$= \underline{8.5\mathbf{i} + 23\mathbf{j}}$ — A1 (2)

(d) At 1400 $\mathbf{s} = 11\mathbf{i} + 17\mathbf{j}$ — M1 A1

At time t, $\mathbf{s} = \underline{11\mathbf{i} + (17 + 5t)\mathbf{j}}$ — M1 A1 (4)

(e) East of $R \Rightarrow 17 + 5t = 23$ — M1

$\Rightarrow t = 6/5 \Rightarrow \underline{1512 \text{ hours}}$ — A1 (2)

(f) At 1600 $\mathbf{s} = 11\mathbf{i} + 27\mathbf{j}$

$\mathbf{s} - \mathbf{r} = 2.5\mathbf{i} + 4\mathbf{j}$ — M1

Distance $= \sqrt{(2.5^2 + 4^2)} \approx \underline{4.72 \text{ km}}$ — M1 A1 (3)

(a) M1 needs square, add and √ correct components

(b) M1 for finding acute angle = arctan (2.5/6) or arctan (6/2.5) (i.e. 67°/23°). Accept answer as AWRT 337.

(c) M1 needs non-zero initial p.v. used + 'their 3' x velocity vector

(d) Allow 1st M1 even if non-zero initial p.v. not used here

(e) A1 is for answer as a time of the day

(f) 1st M1 for using $t = 2$ or 4 (but *not* 200, 400, 6, 16 etc) and forming $\mathbf{s} - \mathbf{r}$ *or* $\mathbf{r} - \mathbf{s}$

Q5, (Jan 2011, Q4)

(a)	speed $= \sqrt{2^2 + (-5)^2}$ $= \sqrt{29} = 5.4$ or better	M1 A1 (2)
(b)	$((7\mathbf{i}+10\mathbf{j})-(2\mathbf{i}-5\mathbf{j}))/5$ $=(5\mathbf{i}+15\mathbf{j})/5=\mathbf{i}+3\mathbf{j}$ $\mathbf{F}=m\mathbf{a}=2(\mathbf{i}+3\mathbf{j})=2\mathbf{i}+6\mathbf{j}$	M1 A1 A1 DM1 A1ft (5)
(c)	$\mathbf{v}=\mathbf{u}+\mathbf{a}t=(2\mathbf{i}-5\mathbf{j})+(\mathbf{i}+3\mathbf{j})t$ $(-5+3t)\mathbf{j}$ Parallel to i $\Rightarrow$ $-5 + 3t = 0$ $t = 5/3$	M1 A1 M1 A1 (4) **[11]**

Q6, (Jun 2007, Q7)

(a)	$\mathbf{v}=\dfrac{8\mathbf{i}+11\mathbf{j}-(3\mathbf{i}-4\mathbf{j})}{2.5}$ or any equivalent $\mathbf{v}=2\mathbf{i}+6\mathbf{j}$	M1 A1 A1 **(3)**
(b)	$\mathbf{b}=3\mathbf{i}-4\mathbf{j}+\mathbf{v}t$ ft their $\mathbf{v}$ $=3\mathbf{i}-4\mathbf{j}+(2\mathbf{i}+6\mathbf{j})t$	M1 A1 ft A1cao **(3)**
(c)	**i** component: $-9+6t=3+2t$ $t=3$	M1 M1 A1
	j component: $20+3\lambda=-4+18$ $\lambda=-2$	M1 A1 **(5)**
(d)	$v_B=\sqrt{(2^2+6^2)}$ or $v_C=\sqrt{(6^2+(-2)^2)}$	M1
	Both correct	A1
	The speeds of B and C are the same cso	A1 **(3)** **[14]**

Q7, (Jun 2013(R), Q6)

(a)	Use of $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$	M1
	$(-4\mathbf{i}+2\mathbf{j})+(3\mathbf{i}+3\mathbf{j})t = (-4+3t)\mathbf{i}+(2+3t)\mathbf{j}$	A1
		(2)
(b)	$(6\mathbf{i}+\mathbf{j})+(-2\mathbf{i}+n\mathbf{j})t = (6-2t)\mathbf{i}+(1+nt)\mathbf{j}$	B1
	Position vectors identical $\Rightarrow -4+3t = 6-2t$ **AND** $5t = 10$,	M1
	Either equation	A1
	$2+3\times 2 = 1+2n$,	**DM1**
	$n = 3.5$	A1
		(5)
(c)	Position vector of P is $(-4+6)\mathbf{i}+(2+6)\mathbf{j} = 2\mathbf{i}+8\mathbf{j}$	M1A1
	Distance OP $= \sqrt{2^2+8^2} = \sqrt{68} = 8.25$ (km)	M1A1
		(4)
		[11]

Q8, (Jun 2013, Q7)

(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2+(-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i}+3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1-2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t-3) = -3(1-2t)$	M1 A1
	Solving for t	**DM1**
	t = 2/3, 0.67 or better	A1
		(6)
		[11]

Notes for Question 7

Q7(a)	B1 for **i** – 3**j**. M1 for √ (sum of squares of cpt.s) A1 for √10, 3.2 or better
Q7(b)	M1 for clear attempt to sub $t = 2$ into given expression. A1 for 315.
Q7(c)	**(i)** First M1 for $1 - 2t = 0$. First A1 for $t = 0.5$. N.B. If they offer two solutions, by equating both the **i** and **j** components to zero, give M0. **(ii)** First M1 for $\frac{1-2t}{3t-3} = \pm(\frac{-1}{-3})$ o.e. (Must be an equation in t *only*) First A1 for a correct equation (the + sign) Second M1, dependent on first M1, for solving for t. Second A1 for 2/3, 0.67 or better.