

Wilcoxon Rank-Sum Tests (From OCR 4735)

Q1, (Jun 2008, Q4)

<p>--- (ii) $H_0: m_M = m_A, H_1: m_M \neq m_A$ "average" $R_m = 40, m(m+n+1) - R_m = 72$ $W = 40$ CR: $W \leq 38$ 40 not in CR, so do not reject H_0 Insufficient evidence that median times differ</p>	<p>B1 M1 A1 B1 M1 A1</p>	<p>Both hypotheses, AEF. Not Both found A0 if no or wrong 72 Or equivalent In context. B1 if no M1 but conclusion correct Allow average here</p>
	6 (7)	

Q2, (Jun 2010, Q5)

(i)	Assumes salaries symmetrically distributed $H_0: m(\text{edian}) = 19.5, H_1: m(\text{edian}) \neq 19.5$ $P = 867$ (or 408) Using normal approximation $\mu = \frac{1}{4} \times 50 \times 51 (= 637.5)$ $\sigma^2 = 50 \times 51 \times 101 / 24 (= 10731.25)$ $z = (a - 637.5) / \sqrt{10731.25}$ Use $a = 866.5$ $= 2.211, \text{ or } 2.215 \text{ or } 2.220$ (– from 408) Compare their z with 1.96 and reject H_0 There is sufficient evidence at the 5% SL that the median salary differs from £19 500	B1 B1 M1 A1 A1 M1 A1 A1 M1 A1 ft 10	In context For both ; not μ ; accept words $a=866.5, 867, 867.5$ (or 408.5, 408, 407.5) Or p -value rounding to 0.026 or 0.027 Compare with 0.05 or equivalent ft z Or find critical region
	(ii)	Use sign test when salary distribution is skewed	B1 (11)

Q3, (Jun 2011, Q5)

(i)	Does not require a known probability distribution	B1 B1 M1 M1 A1 B1 M1 A1 A1 M1 B1 A1 [11]	Or equivalent Medians Use $N(39,39)$ with cc B1 $P(W \leq 27.5), Z = -1.84$ or equivalent M1 Not in CR etc A1 $P(W \leq 27.5) = -2.07$ M1A1 In CR H_0 rejected A1
(ii)	$H_0: m_A = m_B, H_1: m_A \neq m_B$ Ranks: A 1 2 3 5 6 10 B 4 7 8 9 11 12 $R_A = 27, 78 - 27 = 51, \text{ so } W = 27$ OR: $R_B = 51, 78 - 51 = 27$ 5% CV = 26 $27 > CV$ so do not reject H_0 there is insufficient evidence at the 5% SL to indicate a difference in breaking strengths	B1 B1 M1 M1 A1 B1 M1 A1 A1 M1 B1 A1 [11]	Or equivalent Medians Use $N(39,39)$ with cc B1 $P(W \leq 27.5), Z = -1.84$ or equivalent M1 Not in CR etc A1 $P(W \leq 27.5) = -2.07$ M1A1 In CR H_0 rejected A1
(iii)	B would have an extra rank 13 W still 27 but CV now 27 H_0 is now rejected	B1 B1 M1 M1 A1 B1 M1 A1 A1 M1 B1 A1 [11]	Or equivalent Medians Use $N(39,39)$ with cc B1 $P(W \leq 27.5), Z = -1.84$ or equivalent M1 Not in CR etc A1 $P(W \leq 27.5) = -2.07$ M1A1 In CR H_0 rejected A1

Q4, (Jun 2014, Q6)

(i)	$1 + 2 + \dots + 11$ $= 66$	M1 A1 [2]	M0 if followed by incorrect work.	
(ii)	$\frac{(N) (132,264)}{(W + 0.5 - "132")}$ $\sqrt{"264"}$ $< -$ 2.576 Solve inequality $< 89.6 \quad (66 \leq) W \leq 89$	B1 M1 M1* B1 *M1 A1 [6]	Allow wrong, or no, cc. May be earned later. Allow 2.58 or equation if final answer uses < or ≤ Integer needed.	Allow reversed if consistent OR $132(-0.5) \pm z \times \sqrt{264}$ M1 $z = 2.576$ or 2.58 B1 (89.6, [173.4]) A1 < lower limit M1 ≤ 89 A1 Allow if lower limit only considered.

Q5, (Jun 2012, Q3i)

(i)	Populations have identical/same distributions (apart from location) $(H_0: m_1 = m_2, H_1: m_1 \neq m_2)$ Ranks 1 2 4 6 9 10 3 5 7 8 11 12 13 $R_m = 32, m(m + n + 1) - R_m = 52$ $W = 32$ Critical value = 29 $32 > 29$, do not reject H There is insufficient evidence at the 10% significance level of a difference between the median marks of the two groups. oe.	B1 M1 A1 A1 B1 M1 A1 [7]	Allow 'Data quantitative' Can be implied. M1A0A1 possible Correct first conclusion ft TS and CV ft TS only.	Allow 'No assumption necessary' stated.
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Q6, (Jun 2013, Q4)

(i)	Distribution of heights may not be normal/is unknown	B1 [1]	Allow “No assumption required”, but nothing else Not “groups independent” unless something else as well
(ii)	$H_0: m_A = m_B, H_1: m_A \neq m_B$ Ranks: A: 4, 8, 10, 11, 14, 15, 16, 18, 20, 21, 22 B: 1, 2, 3, 5, 6, 7, 9, 12, 13, 17, 19 $m = n = 11, R_m = 159$ or 94 Use normal approximation with mean 126.5 [= 253/2] Variance 231.92 [= 2783/12] (α) $P(\leq 94) = \Phi((94.5 - 126.5)/\sqrt{(231.92)})$ or $P(\geq 159) = 0.0178$ < 0.025 and reject H_0 <hr/> (β) $z = (94.5 - 126.5)/\sqrt{(231.92)} = -2.101$ < -1.96 so reject H_0 There is evidence that salinity affects growth	B1 B1 B1 M1 B1 M1 A1 M1 M1A1 M1 A1 [9]	Medians. Allow words in context. Not μ unless “median” stated allow $\frac{1}{2} \times 11 \times (11 + 11 + 1)$ allow $\frac{1}{12} \times 11 \times 11 \times (11 + 11 + 1)$ Standardising. Allow no/incorrect cc. Value ft TS <hr/> Standardising ; value ft TS Or equivalent in context. ft TS.

Q7, (Jun 2015, Q6)

<p>H₀:The samples are drawn from identical popns. H₁:The samples are from different popns.</p> <p>Mean=188.5 Var=471.25</p>	<p>B1 B1 B1</p>	<p>Allow $m_1=m_2$; $m_1 \neq m_2$</p> <p>Allow 13x29/2 Allow 13x15x29/12</p>	<p>Critical region method . First B1B1B1as main scheme $\frac{x+0.5-188.5}{\sqrt{471.25}} = \text{or} < -1.96$ M1A1B1 $x < 146$ A1 135 is in CR, rej H₀ M1 Conclusion A1</p>
<p>$\frac{135 + 0.5 - "188.5"}{\sqrt{471.25}}$ -2.44 CV=-1.96 TS<CV, reject H₀ Sufficient evidence that the samples were drawn from different populations.</p>	<p>M1A1 ft A1 B1 M1 A1 [9]</p>	<p>Allow M1A0 for missing or incorrect c.c.</p> <p>Allow -2.46 no c.c, -2.49 wrong c.c.</p> <p>Ft both TS,CV Not over-assertive. Cwo, allow from(-2.46 or -2.49</p>	<p>0.0073 (or 0.0069 or 0.0064) B1 pft< 2.5% (allow 5% for M1), rej H₀</p>

Q8, (Jun 2016, Q2)

<p>H₀: $m_A = m_B$, H₁: $m_B < m_A$ Attempt ranks 15, 1, 6, 12, 11, 13, 14; 7, 9, 3, 10, 8, 2, 5, 4 $R_m = 72$ $W = 40$ CV = 41 "40" < 41 reject H₀ Evidence that treatment B is more effective.</p>	<p>B1 M1 A1 A1 A1 B1 M1 A1 [8]</p>	<p>For both. Allow any sensible hypotheses.</p> <p>Ft TS and CV. In context, not over-assertive. Cwo.</p>	
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